EUREKA

THE ARCHIMEDEANS' JOURNAL

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THE JOURNAL OF THE ARCHIMEDEANS

(The Cambridge University Mathematical Society: Junior Branch of the Mathematical Association.)

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Editorial

"We have in this issue (No. 3) for the first, but not, we hope, the last time, articles written by Senior Members of the University"—thus wrote some of our predecessors, less than eighteen months ago. Their hopes have certainly been fulfilled, but hardly in the manner they intended; for the number of articles submitted to EUREKA by undergraduates has steadily decreased ever since, until now the editors feel almost like writing—"we have in this issue, apparently for the last time, articles written by Junior Members of the University." Let us make ourselves clear. We are not complaining of the *increase* in articles by the Senior Members (indeed we feel honoured that so many of them are willing to write for us), but of the fact that it is necessary to ask them to write the articles that you should be writing. EUREKA was, after all, started in order to give you an opportunity to express and discuss your ideas, and to publish the results of your researches, however unorthodox or immature, and it seems a pity that it should fail in its main object, that of expressing undergraduate opinion.

Incidentally, do you doodle? If so, let your subconscious mind solve a problem for the Archimedeans. It has been suggested that the University crest on the front of the Archimedeans' card should be replaced by a distinctive crest for the private use of the Archimedeans, both on the card and, possibly, on an Archimedeans' tie. Any-one with bright ideas should produce a sketch (preferably not on exam. blotting paper, or on the back of an old envelope) and give it to his college rep. or to any member of the committee. No prizes are awarded.

Any contributions, complaints, comments or suggestions should be addressed to the Editor of EUREKA, The Archimedeans, c/o the Mathematical Faculty Library, New Museums, Cambridge (as soon as you like, but not later than 1st December, 1941).

Archimedeans' Activities

Besides those which are described more fully below, the Archimedeans held two other evening meetings last term. These consisted of talks on "The Non-Spinning Gyro," by Dr. G. F. C. Searle, and "The Mathematical Theory of Statistics," by Prof. H. Simpson. There were also two tea-time meetings, and various meetings of the three groups, Music, Chess and Bridge. This term activity has been on a very small scale, the only three meetings of the Society or its groups being the annual business meeting, a meeting organised by Queen Mary College Mathematical Society (in which we were asked to join), when Mr. Soal spoke about Modern Psychical Research, and a solitary meeting of the Music Group.

"LANDMARKS IN NATURAL PHILOSOPHY"

The third meeting of the Lent term was an account by Mr. Cunningham of the historical development of natural philosophy, illustrated with some interesting slides, many of original manuscripts of the founders of this subject.

First in the field were the Greek mathematicians, principally Archimedes and Ptolemy; the former's work on hydrostatics and similar problems is too well known to recount, while the latter prepared extremely accurate tables of trigonometrical and astronomical data which were of great use in mensuration and navigation at that time.

The more complete form of astronomy as known to us, was founded principally by Kepler, who followed on the work of Copernicus and Tycho Brahe. The development of this particular branch was caused by a tremendous increase in overseas trade, with its problems of accurate navigation. About the same time Galileo was led, after the use of gunpowder, to investigate the motion of projectiles—the first development of mechanics. This previous work was all firmly founded and further developed by Newton, in his classical theory of mechanics and gravitation. From Newton's theory, John Couch Adams, after prolonged observation of the perturbations of Uranus, was able to predict the existence of Neptune, which was soon confirmed.

The main landmarks in the more modern era were in connection with electrical phenomena. Faraday and his pupil Maxwell investigated and expressed mathematically the fundamentals of this subject. Maxwell, however, was dissatisfied with the classical theory and Lorentz completed his ideas, leading to what is now known as the Einstein-Lorentz transformation. From this point, expressing the fact that any physical law holds good independently of one's co-ordinate axes, Einstein developed his special theory of relativity and later the general theory, with the help of Riemann Another landmark was Planck's theories of quantisation of energy, of which the only justification then was that they explained a number of physical problems hitherto unaccounted for. From this point, others such as Bohr, Schrödinger, Heisenberg, and Prof. Dirac have brought quantum mechanics to its present stage.

From his account, as the speaker had pointed out previously, it was clear that natural philosophy had not developed *in vacuo*, but was the result of attempts to express the natural phenomena of the real world in precise mathematical form. R. F. B.

THE PROBLEM DRIVE

The problem drive held a year ago was so successful that it was decided to hold another this year, and it took place in St. John's College on Wednesday, March 5th. Sixty competitors divided themselves into pairs and then proceeded to do ten very varied problems. Included in these were tangrams and a problem in three-dimensional noughts-and-crosses (a game that is not played as much as it deserves) as well as the more standard type of problem which may be found in books such as *The Canterbury Puzzles*. The pairs were allowed five minutes for each question, and although this caused a few shudders, it was found to be, in general, quite sufficient.

Two pairs tied for first place, and were then forced to play a game of three-dimensional noughts-and-crosses on the blackboard, aided and encouraged by the spectators. In this, after both sides had narrowly escaped defeat, Garner and Turnbull of St. John's were successful against Tomlinson and Boardman of Clare, and so were awarded the first prizes. Two ladies from Bedford, who would undoubtedly prefer to remain anonymous, had the lowest score, and (poetic justice) were presented with copies of the Penguin *Problem Book*.

All competitors, whether successful or otherwise, appeared to have an enjoyable evening, and we may expect more Problem Drives in future. J. L. K.

MUSIC GROUP

On February 20th, the members of the Group spent a very pleasant hour in Mr. M. H. A. Newman's rooms listening to a violin and piano recital given by Mrs. Allen and Mr. Newman. The programme was:

Sonata in G Minor	*Tartini*
Toccata and Fugue in E Minor	*J. S. Bach*
Sonata in F Major (K 376)	*Mozart*
Three short pieces	*Lenox Berkeley*
Sonata in G Minor	*John Stanley*

The Group is known to have a preference for music of the eighteenth century, and therefore the first three items met with our full approval. To remind us, however, that music is still being written, Mr. Newman introduced to us the music of Lenox Berkeley, whose striking style provoked considerable discussion. The final work, by a little-known contemporary of Arne, brought us back to more familiar ground, and concluded a most enjoyable evening, for which our grateful thanks are due to Mrs. Allen and Mr. Newman.

Three gramophone recitals were also held during the term, the principal works played being the Emperor Concerto by Beethoven, the Trio in B Flat by Schubert, and Sibelius' Second Symphony.

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An Undergraduate Apology

APOLOGIES for themselves and their subjects have been made by two well-established mathematicians but, although one of them emphasised that mathematics was a young man's game, no reference has been made to the young mathematicians of to-day.

To-day young people cannot retire into the protection of the university to pursue their studies; they cannot rely on their brains to retain for them the right to study. The world is at war, and mathematicians have to show that they deserve a place in society. They cannot say, as Professor Hardy does, that even if their lives are wasted they are only a few, and if real mathematics is as remote from physical reality as Professor Hardy claims, then we must stop studying it immediately and turn our attention to something more relevant. Even the theory of numbers which is as pure and remote as mathematics can be, is based on the operation of counting physical objects. Russell's famous definition of number is valuable because it suggests the general principle, in which we believe, that ideas are based solely on our observations of the physical world. Professor Hardy ascribes the importance of certain theorems to their lack of dependence upon physical objects, and he believes that a mathematical reality and truth exists apart from the physical one; but we assert that mathematics is an abstraction from the natural world, and that those who study it idealise and perfect natural objects before they develop theories about them; that the whole of mathematics is founded on the world we experience through our senses. Indeed, the human mind cannot conceive things beyond this material world, because such things have no relevance to the world we live in, which is the same as saying that they do not exist as far as we are concerned.

Having denied that real mathematics deals with truths outside our world, we must be wary of going to the other extreme and claiming that it is an exposition of the truths of nature. We must not follow the example of theologians and claim that our subject is an explanation of the world, for there are already too many men keen to explain this. Mathematics has a value far greater than such idle conceits. Mr. Cunningham wrote in the last issue of EUREKA: "To me the real interest has been the progressive revelation of mathematical order in the natural world." In fact he asserts that mathematics is the discovering of truths quite independent of ourselves, which exist already and which we only discover. We would rather say that mathematicians create them. For, although the objects with which mathematics deals are suggested by objects of the natural world, they are not natural objects. The order which mathematical physicists seem to be discovering refers to idealised, simplified, perfected objects which have been extracted from nature. We do not get a picture of an orderly world, but rather an orderly picture of a world which is usually somewhat like the natural one for practical purposes. If it gets too much like the natural one it is too complicated to contemplate; if it gets very far from it, it is too difficult to imagine; we are dependent for material to study on what we observe, but we do not study what we observe.

Mathematics, then, does not set the world, but the mind of the observer in order, and therein lies its great value. It enables one to understand what one is saying, and to know how assertive one may be. In his article "The Euclidean Spook,"* Mr. Scott posed the question "How can they (children) be expected to reason logically about dry matters like points, lines, and angles when they have never learnt to reason about things in which they are interested?" which immediately makes one ask "How can anyone be expected to reason about ordinary things in which prejudice and emotion play a large part if they cannot reason about dry matters?" Thought on these lines suggests a reason for Mr. Cunningham's belief that more of the marks of a mathematician would make a better world.

It is only a few who, on graduating, continue to lead purely mathematical lives, developing and adding to the store of mathematical knowledge and methods. Though their work in advanced fields is likely to be of practical importance someday that is not the reason for their study. But we are not concerned here with these few, nor with the other few who become technicians and whose place in society is justified by their usefulness. We are concerned with the many whose life is not and ought not to be bound up in mathematics alone, whereas it is for the few and not the many that present arrangements cater. That one is good at,

* In last issue of *Eureka*.

and interested in, mathematics, is sufficient reason for receiving a mathematical education, whatever one intends to do later, because by it one can most fully develop and, as well as being most helpful to others, one can live a life worth while in itself. However, we do not believe that because it suits some it is good for all. It is a crime to compel anyone to specialise in mathematics unwillingly. But at present in deciding whether or not a pupil is to read mathematics, neither his own happiness nor the probability that he will thus become a useful member of society, is taken into account. Whether he can learn enough to pass examinations, whether he is likely to be able to get a good job with his degree—not, we note, whether he will be able to do a job well because of his education—and whether his parents can afford it, are the deciding factors. These are the natural outcomes of society's being based on the profit motive; where the getting of a job is more important than the doing of it; where what one does, and when, and where, overrides the consideration of how to live; where money is the object of, and not a convenience in living, and where competition for possession has ousted co-operation for attainment.

The social order which has brought the present calamities must be changed, and with it the method of education. But before suggesting changes it is as well to see how mathematics gives desirable qualities to those who study it. Writers, with the possible exception of James Joyce, have as their means of expression a language already created for them; composers must write their music in the established notation or no one will play it; but because of the powerful method of definition, the mathematician can build up a language of his own. He can create his own objects, and his own words to describe them and their behaviour. His development is not retarded by a limited range of implements of expression, and honest thought is not kept from him by the vague common usage of his words. Inventiveness, determination to search for truth, the avoidance of dogmatism not founded on certainty, and a clear idea of the meaning of what he says are the qualities he acquires because his thought is free and yet under his control. Modern teaching, however, built around the need for good examination results, encourages the mere acquisition of knowledge. Even if originality is not frowned on officially, there is neither room nor time for it in working for examinations. Technique and ability to make one's own advances are spoiled by having to learn what others have discovered instead of studying with the help of what others have discovered. Most students are obliged to rush madly through a syllabus, skipping details, and hopping over explanations, with the result that they learn but do not study. There is too much concentration on the acquisition of knowledge and too little on the understanding of it.

Students should be given more opportunity to learn to think critically; to understand, not once, but several times and in many ways, each step they make; to contemplate the significance of what they do; above all, to use their technique and ideas in other realms of thought and experience. Mathematics in isolation is of little value to the majority of students—to the prospective teachers and those who will never use their mathematics as such again. They need to understand the interdependence of mathematics and other branches of thought, and the connection between mathematics and the society in which it has developed, in order that their powers of logical reasoning can be of real use to the community. Students should develop, as a result of their study, a keen sense of judgment, not only of mathematics, but of the mass of complex problems that confront the world to-day; they must not be content to accept the ready-made opinions of others.

To achieve this we suggest that the concentration should be less on mathematics as a means of reaching an ultimate goal—the discovery of the nature of the physical world—and more on the methods of mathematics, the deductive reasoning and logical thought which enable those who master them to gain something of value from a mathematical education, and to establish themselves as useful members of society. D. J. T. and R. S. S.

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The Kingdom of Mathematica

By R. C. Lyness

In the kingdom of Mathematica every year a ceremony takes place at which the king rewards his counsellors. The king and the counsellors seat themselves around a circular table. In front of each a sheet of gold exactly 10 inches square is placed with one pair of edges vertical and the other pair horizontal. At a given signal the king draws a horizontal line and a vertical line each from edge to edge of his square of gold. The counsellors then each draw a horizontal and vertical line on their squares, the following rule being observed. The area of the rectangle below the horizontal line and to the left of the vertical line (i.e. the bottom left-hand rectangle) on the square of every person seated at the table must be equal to the area above the horizontal line on his right-hand neighbour's square and equal to the area to the right of the vertical line on his left-hand neighbour's square. Every counsellor is rewarded by being allowed to cut out and retain the bottom left-hand rectangle on his square. At this year's ceremony each counsellor received a whole number of square inches of gold. The king's bottom left-hand rectangle is not a square.

How many square inches did each counsellor get?

The Faking of Genetical Results

By Prof. J. B. S. HALDANE

MY father published a number of papers on blood analysis. In the proofs of one of them the following sentence, or something very like it, occurred: "Unless the blood is very thoroughly faked, it will be found that duplicate determinations rarely agree." Every biochemist will sympathise with this opinion. I may add that the verb 'to lake,' when applied to blood, means to break up the corpuscules so that it becomes transparent.

In genetical work also, duplicates rarely agree unless they are faked. Thus I may mate two brother black mice, both sons of a black father and a white mother, with two white sisters, and one will beget 10 black and 15 white young; the other 15 black and 10 white. To the ingenuous biologist this appears to be a bad agreement. A mathematician will tell him that where the same ratio of black to white is expected in each family, so large a discrepancy (though how best to compare discrepancies is not obvious) will occur in about 26 per cent. of all cases. If the mathematician is a rigorist he will say the same thing a little more accurately in a great many more words.

A biologist who has no mathematical knowledge, and, what is vastly more serious, no scientific honour, will be tempted to fake his results, and say that he got 12 black and 13 white in one family, and 13 black and 12 white in the other. The temptation is generally more subtle. In one of a number of families where equality is expected he gets 19 black and 6 white mice. It looks much more like a ratio of 3 black to 1 white. How is he to explain it? Wasn't that the cage whose door once seemed to be insecurely fastened? Perhaps the female got out for a while or some other mouse got in. Anyway he had better reject the family. The total gives a better fit to expectation if he does so, by the way. Our poor friend has forgotten the binomial theorem. A study of the expansion of $\left(\frac{1+x}{2}\right)^{25}$ would have shown him that as bad a fit or worse would be obtained with a probability of 122703.2^{-23}, or $\cdot 0146$. There is nothing at all surprising in getting one family as aberrant as this in a set of 20. But he is now on a slippery slope.

He gets his Ph.D. He wants a fellowship, and time is short. But he has been reading *Nature* and noticed two letters* to that journal of which I was joint author, in which I might appear to have hinted at faking by my genetical colleagues. Thoroughly alarmed, he goes to a venal mathematician. Cambridge is full of

* U. Philip and J. B. S. Haldane (1939). *Nature*, 143, p. 334.
Hans Grüneberg and J. B. S. Haldane (1940). *Nature*, 145, p. 704.

mathematicians who have been so corrupted by quantum mechanics that they use series which are clearly divergent, and not even proved to be summable. Interrupting such a one in the midst of an orgy of Bhabha and benzedrine, our villain asks for a treatise on faking. "I am trying to reconcile Milne, Born, and Dirac, not to mention some facts which don't seem to agree with any of them, or with Eddington," replies the debauchee, "and I feel discontinuous in every interval; but here goes."

"I suppose you know the hypothesis you want to prove. It wouldn't be a bad thing to grow a few mice or flies or parrots or cucumbers or whatever you're supposed to be working on, to see if your hypothesis is anywhere near the facts. Suppose in a given series of families you expect to get four classes of hedgehogs or whatnot with frequencies p_1, p_2, p_3, p_4, and your total is S, I shouldn't advise you to say you got just Sp_1, Sp_3, Sp_3, and Sp_4, or even the nearest whole number. Here is what you'd better do. Say you got A_1, A_2, A_3 and A_4, and evaluate

$$\chi^2 = \frac{(A_1 - Sp_1)^2}{Sp_1} + \frac{(A_2 - Sp_2)^2}{Sp_2} + \dots$$

Your χ^2 has three degrees of freedom. That is to say you can say you got A_1 red, A_2 green, and A_3 blue hedgehogs. But you will then have to say you got $S - A_1 - A_2 - A_3$ purple ones. Hence the expected value of χ^2 is 3, and its standard error is $\sqrt{6}$; so choose your A's so as to give a χ^2 anywhere between about 1 and 6. This is called faking of the first order. It isn't really necessary. You might have $p_1 = \frac{9}{16}$, $p_2 = \frac{3}{16}$, $p_4 = \frac{1}{16}$, and $A_1 = 9$, $A_2 = A_3 = 3$, $A_4 = 1$. The probability of getting this is $\frac{16!\ 3^{24}}{9!\ (3!)^2\ 1!\ 16^{16}}$, which is only just under ·04. However, it looks better not to get the exact numbers expected, and if you do it on a population of hundreds or thousands you may be caught out.

Your second order faking is the same sort of thing. Supposing your total is made up of n families, and you say the rth consisted of a_{r1}, a_{r2}, a_{r3}, a_{r4} members of the four classes, s_r in all, you take

$$\frac{(a_{r1} - s_r p_1)^2}{s_r p_1} + \frac{(a_{r2}{}^2 - s_r p_2)^2}{s_r p_2} + \dots$$

and sum for all values of r. Your total ought to be somewhere near $3n$. The standard error is $\sqrt{6n}$, and it's better to be too high than too low. A chap called Moewus in Berlin who counted different types of algae (or so he said), got such a magnificent agreement between observed and theoretical results, that if every member of the human race had repeated his work once a month for 10^{12} years, they might expect as good a fit on one occasion (though not with great confidence). So Moewus certainly hadn't done any second order faking. Of course I don't suggest that he did any faking at all. He may have run into one of those theoretically possible miracles, like the monkey typing out the text of Hamlet by mere luck. But I shouldn't have a miracle like that in your fellowship dissertation.

There is also third order faking. The $4n$ different components of χ^2 should be distributed round their mean in the proper way. That is to say, not merely their mean, but their mean square, cube, and so on, should be near the expected values (but not too near). But I shouldn't worry too much about the higher orders. The only examiner who is likely to spot that you haven't done them is Haldane, and he'll probably be interned as a Red before you send your thesis in. Of course you might get R. A. Fisher, which would be quite as bad. So if you are worried about it you'd better come back and see me later."

Man is an orderly animal. He finds it very hard to imitate the disorder of Nature. In fact the situation is the exact opposite of what the reader of Paley's *Evidences* might expect. But the problem is an interesting one, because it raises in a sharp and concrete way the question of what is meant by randomness, a question which, I believe, has not been fully worked out. The number of independent numerical criteria of randomness which can be applied increases with the number of observations, but much more slowly, perhaps as its logarithm. The criteria now in use have been developed to search for excessive irregularity, that is to say, unduly bad fit between observation and hypothesis. It does not follow that they are so well adapted to a search for an unduly good fit. Here, I believe, is a real problem for students of probability. Its solution might lead to a better set of axioms for that very far from rigorous but none the less fascinating branch of mathematics.

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Solutions of Problems

MATHEMATICIANS IN THE ARMY.

The major was in college with the bombardier, and the gunner had read mathematics.

CIPHERS.

(i) The enemy attacked our position this morning at dawn. They used tanks aeroplanes and gas. We repelled the attack but received many casualties. We need reinforcements.

(ii) The general's orders were that the division would attack the enemy; but the enemy knew this and they attacked first.

ROUND THE TABLE.

From left to right starting from the empty chair sat Mr. Black wearing a purple tie and brown socks who had a blue car, Mr. Green with brown tie and black socks who owned a grey car, Mr. Brown with grey tie and blue socks, owner of a purple car, Mr. White wearing blue tie and green socks, owner of a brown car, Mr. Purple with black tie and white socks, owner of a green car, Mr. Blue wearing white tie and grey socks, owner of a black car, and Mr. Gray with a green tie and purple socks who owned a white car.

PICCADILLY UNDERGROUND STATION.

Of the 128 people at Piccadilly, 104 either had used or were about to use the Bakerloo Line. Hence the answer required is 80.

CROSSWORDS.

Across.—A: 11↊2·01. B: ↊0021. C: 100↋202. D: ƖƖ103. E: ↊1↊. F: ↋0031.

Down.—A: 1Ɩ1↊2·↋. G: ↊↊0Ɩ10. H: 20↋1. J: 002Ɩ↊1. K: 123↊0.

Up.—L: 10021.

Across.—1: 1331. 3: 9863. 5: 917. 7: 86507. 9: 430. 11: 375. 13: 5782435. 14: 775. 15: 273. 17: 14641. 20: 608. 21: 2325. 22: 2747.

Down.—1: 1374. 2: 196. 3: 970. 4: 3275. 6: 1532160. 7: 80751. 8: 73321. 10: 357. 12: 757. 14: 7912. 16: 3267. 18: 465. 19: 482.

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Geodesia

By J. G. OLDROYD

IN Geodesia there are no hills and every road is the shortest possible route connecting two towns. Unfortunately for the traveller, however, the signposts at road junctions do not indicate distances, but only give the angles between the various roads.

A visitor was being conducted by car along the frontiers of the State, and the only information he could obtain was from these signposts. He made the following notes during the tour: "Turned right through 86° 17′; left through 45° 6′; right through 20° 42′; right 30° 17′; left 19° 18′; left 92° 38′; left 20° 41′; right 13° 5′; left 62° 7′; left 72° 8′; right 30°; left 60° 37′; left 20° 3′; left 60° 27′; right 93° 43′." Finally, he found himself leaving the country by the same road as he had entered.

What could he say about the size of the country?

Reform of School Mathematics

By A. Robson

School mathematics may be divided into two parts. There is first an elementary course, which ends for the abler pupil at the age of 14 or 15, in which the future mathematician is not segregated from his less fortunate contemporaries. Then there is a sixth-form course in which the budding mathematician works apart, or with the would-be engineers and scientists. Readers of Eureka will naturally be more interested in the second stage, although, on the principle of the greatest good of the greatest number, the reforms of the earlier stage have been the more important.

Until 1903 the teaching of elementary geometry was dominated by the regulations of examinations like the Little Go which required that propositions should be proved by Euclid's methods or at least by methods which were consistent with the order in which Euclid arranged the propositions two thousand years ago. The thirteen books of Euclid's *Elements* contain over 450 propositions of which about 150 deal with two-dimensional geometry. The modern editions of Euclid which were in use in English schools at the beginning of the century contained most of the 150, together with supplementary results and riders. The average student learnt some of the propositions and failed over most of the riders. In extreme cases those propositions likely to occur in examinations were learnt by heart. Thus there is the story of the undergraduate who was aggrieved because he was ploughed in the Little Go although he had correctly reproduced eight propositions out of ten; it turned out that he had put the letters in his diagrams at the wrong corners.

Even when the regulations were changed, reforms were only introduced gradually. For one reason, the teachers had been brought up in the old style, and many of them had still to be convinced that the new was better. The first noticeable effect was the introduction of numerical and practical work and the use of instruments such as set-squares and protractors. The harder riders were replaced or supplemented by easier exercises within the capacity of all but a few. Gradually it has come about that geometry bookwork is less emphasised, and students are now expected to do something (however modest) for themselves. This in itself is a great gain. But there has been a more important change. Euclid's proofs were strictly geometrical; there was no question of using algebra or trigonometry in geometry. You were lucky if your teacher allowed you to use a minus sign or to write AB^2 for the square on AB. And it was not only in geometry that this kind of restriction was imposed; in arithmetic, algebra was forbidden; trigonometry was regarded as outside the course. Nowadays the aim is to treat mathematics as a single subject, each part of which is available to help or illustrate the other parts. In the process of fusion of the subjects, the graph has played a very large part. The idea of a function, from a graphical point of view, the ideas of differential and integral calculus, and the use of coordinates, are now included, at least for the abler pupils, in the elementary course. Further reforms are of course needed; the percentage of pupils who can be given some insight into such subjects ought to be steadily increased.

In the sixth-form course also, mathematics has suffered in the past from division into watertight compartments. The calculus was formerly left until too late. In the entrance scholarship examinations at Cambridge integral calculus was included for the first time in 1906. At that time freshman scholars, although they might be expert at the processes of differential calculus, were usually quite ignorant of the fundamental principles of that subject. This was partly the fault of text-books; but also the curriculum was unbalanced. Too much algebra and trigonometry was attempted, considering that so little calculus was done. The fusion of certain parts of these three subjects has been an important reform. There were suggestive articles about the fundamental theorems of analytical trigonometry in the *Mathematical Gazette* of 1904 over the signatures of T. J. I'A. Bromwich and G. H. Hardy, but probably the improvements that have been made in the teaching of analysis were due largely to the appearance of Hardy's *Pure Mathematics* in 1908. This book also made it clear what should be the attitude to complex numbers.

Geometry suffered much from the compartment theory. After a course in "geometrical conics" in which the metrical properties of conics were deduced from the focus-directrix definition by the methods of Euclidean geometry, there followed a course of "analytical conics" in which many of the same results were proved again by cartesian methods. The subject taught was "conics" rather than "geometry." Important methods of geometry, such as the use of determinants, parameters, and line co-ordinates and the idea of duality, were omitted or postponed. Even when homogeneous co-ordinates and semi-projective geometry were started, the metrical point of view still predominated; even now it is not clear what should be the school approach to projective geometry. In 1907, G. H. Hardy wrote as follows in the *Mathematical Gazette*: "Can anyone tell me of an English book which contains a clear and intelligible account of the line at infinity? . . . most undergraduates seem to believe that there really are points at infinity and that they really do lie on a line, and that if you could get there, you would find that $1 = 0$." Probably there was no satisfactory reply given to this question, and the account given in an appendix added to *Pure Mathematics* met the need.

The want of calculus showed itself in some parts of Geometry, but more noticeably in Dynamics. Forces and accelerations were nearly always constant; differential equations were left for the university. Consequently rather artificial problems had to be devised whose chief merit was that they were solvable by elementary methods.

In the sixth-form work, as in the earlier stages, some time is saved by the omission of unnecessary elaborations, but there is no general agreement about which subjects can be regarded as obsolete or how their places should be filled. We may hazard the guess that a writer in the 100th number of EUREKA will comment on the extraordinary ignorance of freshmen in 1939 of vector analysis, dummy suffixes, and matrices.

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Modern Physical Theory and Mathematics

By M. J. H. MOYAL

THE classical methods of analysis were originally developed to meet the needs and solve the problems of Newtonian mechanics, and the time is not yet far past when physicists still believed they could solve practically all their problems with continuous functions and the theory of linear differential and partial differential equations. The first revolution in modern physical theory occurred with the introduction of statistical and probability notions in connection with the development of the kinetic theory of gases and statistical mechanics. Nevertheless, the mathematical discipline for these was mostly taken over from the analytic mechanics of Lagrange, Hamilton and Jacobi, and only a few elementary notions were borrowed from the calculus of probabilities. The next great step was contained in Einstein's special and general relativity theories; these involved fundamental changes in our ideas of space and time, and for the latter, the application to the universe of Riemannian instead of Euclidean geometry, but the analytic methods were not much changed.

It is with the development of quantum mechanics during the last fifteen years that the need for fundamentally new mathematical methods first appeared. Not only was a readjustment of our philosophical ideas made necessary (for example by Heisenberg's principle of uncertainty) and not only were new physical principles involved, but the consistent and systematic development of the theory by Dirac, Weyl and others utilised mathematical methods—such as non-commutative algebra, operator and group theory—which were radically different from those of the old mechanics.

At the present moment, both relativity and quantum mechanics appear to have come to a dead-end. In spite of numerous attempts, there is no completely satisfactory relativistic theory of the electromagnetic field; in quantum mechanics, too, field theories are not very successful, and great difficulties are encountered in the theory of the nucleus and of the interior of elementary particles. There seems to be widespread opinion that more fundamental changes are still required before these difficulties can be removed, and it is therefore not idle to speculate on the direction in which these changes could be effected. There seem to be three main possibilities; the discovery of new physical principles, more drastic revision of our epistemological notions, or the introduction of new mathematical methods.

It appears to me that we already have the guiding physical principles, and the main elements for a critical revision of our philosophical ideas, and it is therefore my opinion that the most hopeful avenue of development is the third: namely, the discovery of more appropriate mathematical methods. On studying, for example, Einstein's new unitary field theory, one obtains the distinct impression that the difficulties are chiefly mathematical. Quantum mechanics forms an elegant and consistent theory from the mathematical point of view, but a certain arbitrariness in its physical interpretation leads one to wonder whether other methods might not prove more suitable.

One avenue of approach might be through the calculus of probabilities and mathematical statistics. Considering the importance of statistical interpretations in modern physics, it is surprising to notice how little use has been made of the resources of these two branches of mathematics, both of which have made such tremendous progress in the last three decades. To give one example, there have been only isolated and half-hearted attempts to introduce the notion of probability dependence or correlation in physics; the statistical variates (or random variables) usually considered are either independent (e.g. the velocity components of a Brownian particle) or functionally dependent. One attempt in this direction has been made by the author in collaboration with Ph. Wehrle and G. Dedebant to apply the newly developed theory of random functions.

There are other branches of mathematics whose application to physics might prove fruitful, the theory of discontinuous functions for one. On the other hand, it may prove necessary to invent entirely new mathematical methods; we might need for example, a "discontinuous" geometry, where the notion of distance is "atomized," in connection with the discovery by Landau and Peierls of the necessity for an uncertainty on the *position alone* of an electron in relativistic quantum mechanics (Heisenberg's now classical relation applies to the *product* of the uncertainties on conjugate co-ordinates and momenta).

For the last two thousand years, physics and mathematics have been closely connected in their development, new mathematical methods arising from the necessity of solving certain physical problems, new branches of pure mathematics inspiring the solution of certain other problems. The latter process was apparent in the development of both relativity and quantum mechanics. However, the moment has perhaps come for the physicist to turn once more to the mathematician and ask him for new tools with which to pursue his investigations.

▪ ▪ ▪

Extensions of the Playfair Cipher

By F. T. M. Smith

One of the best ways of suppressing alphabetic frequencies in a cipher is the method of polygraphic substitution. A simple example, in which the substitution is digraphic, is the Playfair Square cipher. This is described in the new edition of W. W. Rouse Ball's *Mathematical Recreations and Essays,* and also in *Have His Carcase* by Dorothy L. Sayers, but I will give a brief outline of it here. The order of the alphabet in the cells is determined by the keyword (*see* diagram).

The plain-text is split into pairs of letters. In general, each pair determines a rectangle. The two letters at the remaining corners of the rectangle form the corresponding element of the cipher. Thus the word "Eureka" is split up into EU RE KA and ciphered as BW HB DC. The vertical direction determines the order—thus EU gives BW, but UE gives WB.

Though this method suppresses the frequencies of individual letters, the digraph frequencies remain, forming a guide by means of which the cipher may be solved fairly easily. These digraph frequencies may be eliminated by first applying a transposition-cipher to the plain-text. A simple method is to write the message in lines one above the other, and to pair off any letter in the $2n$th line with the letter immediately above it.

Even with this modification, the cipher has many defects. Special rules must be used when the two letters of a pair are the same, or occur in the same row or column of the square. These give rise to peculiarities which facilitate solution.

To see how the extensions arise, consider the rows and columns to be numbered, so that each letter has "co-ordinates." Thus, in the example given, KA is (5,3)(2,4). The corresponding pair DC is (2,3)(5,4). In general, $(p, q)(r, s)$ gives $(r, q)(p, s)$, where $p \neq r$ and $q \neq s$.

Obviously, this is not the only transformation possible. Thus we might use $(p, q)(r, s) \rightarrow (p, r)(q, s)$. Further, we see there is no need to split the message into pairs of letters at all—we might use such a transformation as

$$\ldots.(p,\ q)(r,\ s)(t,\ u)\ldots. \rightarrow \ldots.p)(q,\ r)(s,\ t)(u,\ldots.$$

For still higher security we write the co-ordinates above each other in lines, thus:

$$\begin{array}{l}\ldots.(p,\ q)(r,\ldots. \\ \ldots.\alpha)(\beta,\gamma)\ldots.\end{array}$$

By pairing off vertically, we obtain $\ldots.(p,\ \alpha)(q,\ \beta)(r,\ \gamma)\ldots.$ This entirely suppresses both alphabetic and all polygraphic frequencies, and thus makes solution extremely difficult.

Nor need we restrict the number of co-ordinates to two. For a three-co-ordinate system, we may either construct a "key-cube," or, alternatively, write down the alphabet in an order determined by the key-word, and number the letters to the radix 3, going from 000 to 222. An extra sign (e.g. "&") is inserted to bring the alphabet up to 27. In the example given below, the key-word is "polygraphic," and the order of the alphabet is polygrahic-bdefjkmnqstuvwxz&.

EXAMPLE OF THE 3-CO-ORDINATE EXTENSION:

Message: "Extension ciphers."

Working—

(1 1 0)(2 2 0)(2 0 2)(1 1 0)(1 2 2)(2
0 1)(0 2 2) 0 0 1 1 2 2 1 0 0 0 2
2 0 0 0 0 2 1 1 1 0 0 1 2 2 0 1

Cipher—

D E P X X L S G V K K G D T Q Z

DIAGRAM:
THE PLAYFAIR SQUARE

P	O	L	Y	G
R	A	H	IJ	C
B	D	E	F	K
M	N	Q	S	T
U	V	W	X	Z

Many other variations are possible, e.g. that in which each letter has five co-ordinates. We number to the radix 2, and as $2^5 = 32$, 6 extra signs (e.g. the figures 2 to 7) are needed. This, however, is rather clumsy, and the gain in security is probably slight.

The general method may be summarised thus: each letter is made to correspond to a combination of numbers. The letters of the plain-text are replaced by the corresponding combinations. The numbers are re-grouped according to some rule. The new combinations are replaced by the corresponding letters. In deciphering, the procedure is reversed.

Can any reader suggest a method of solution?

Other types* of polygraphic substitution depend on linear transformations and matrices in a finite algebra, but these are more complicated and are designed for use only with cipher-machines.

* See the *American Mathematical Monthly*, 1931, Vol. xxxviii, pp. 135–154.

Analysis

By M. L. Cartwright

The title is rather vague; the dictionary says that to analyse is to "examine minutely the constitution of," or "to shew the essence of." To say Modern Analysis is no help, partly because the subject is even less well defined now than it was some years ago, and partly because it suggests a text book which is no longer truly modern, whereas I wish to discuss the subject as it has appeared to me in recent years. One analyst told me that he was interested in anything involving a limiting process, thus excluding a certain amount of algebra, geometry and theory of numbers; the subject certainly includes the theory of series, differential and integral equations and calculus of variations; but the major part of it is concerned with the theory of functions of real or complex variables in one form or another. It melts through the theory of functions of a complex variable into the analytic theory of numbers, and through the theory of functions of real variables into abstract spaces, functional analysis and general topology. It might be more appropriate to dissect mathematical analysis into several subjects if it were not that in England the leadership of Hardy and Littlewood has united workers in this field more closely that they would be elsewhere.

The term Hardy–Littlewood analysis aptly describes a type of analysis represented by Titchmarsh's *Theory of Functions*. Professor Hardy himself has described it as the "hard, sharp, narrow" kind as opposed to the "soft, vague, broad" kind of some American and German mathematicians. It aims at sharp results; that is to say theorems in which the hypotheses are just and only just sufficient to ensure the truth of the conclusion; and its highest achievements are usually considered to be a type of theorem which can be stated in a few lines, but can only be proved with difficulty.

In order to illustrate this point I should like to mention two such theorems, neither of them very new, but more readily intelligible than most recent masterpieces. The first is Littlewood's Tauberian theorem, viz., *suppose that* $\Sigma a_n x^n$ *is convergent for* $|x| < 1$, *and that* $\Sigma a_n x^n \to A$ *as* $x \to 1-0$. *Then if* $a_n = O\left(\frac{1}{n}\right)$ *as* $n \to \infty$, Σa_n *converges to the sum* A. In this case the theorem is comparatively trivial if we strengthen the hypothesis on a_n to $na_n \to o$, and it is false if we relax the hypothesis to $a_n = O(\psi(n)/n)$ where $\psi(n)$ is some function which tends to infinity no matter how slowly. In the course of time the proof has been considerably modified and simplified; it is possible now to deduce it from other well known theorems by simple methods, but there is a hard core of difficulty in the theorem with $a_n = O\left(\frac{1}{n}\right)$ as opposed to the one with $na_n \to o$ and this core can only be penetrated by the use of sharp tools in the way of important mathematical ideas and elegant devices.

The other theorem, Picard's theorem, is much older and is not a product of the Hardy–Littlewood school, but it is generally recognised as one of the most important theorems in the theory of functions of a complex variable. It says that *a one-valued analytic function takes every value except perhaps one, an infinity of times in the neighbourhood of an isolated essential singularity.* Here again subsequent work simplified the proof considerably, but it has not become trivial, and the idea has led to a multitude of generalisations each having its own particular interest.

The mathematician to whom this hard, sharp type of problem appeals usually roams over a fairly wide field, and may find his inspiration outside the realms of analysis itself. For instance, much of the early work on series and on the theory of functions of a complex variable was done in order to solve certain problems in the analytic theory of numbers; later it was developed and generalised. Even in recent years pure mathematicians continue to find inspiration in the applied sciences; biological work has led to the study of certain types of integral equation, and practical problems in the adjustment of loud speakers have given rise to much interesting work on non-linear differential equations, to name only two examples. The idea that a certain result may be true is usually suggested by the study of special cases, by analogy, and even, as in the case of non-linear differential equations, by experimental results. The attack on the problem usually includes a thorough investigation of the consequences in the hope of finding a *reductio ad absurdum* argument, as well as direct calculations, and translation of the problem into terms of some other problem. A wide general background is needed, because success is frequently attained by using methods from one branch of analysis to solve the problems in another; but the mathematician who likes his difficulties concentrated on a sharply defined problem tends to avoid very general work such as the theory of functions of severable variables and extremely abstract work, although certain steps in the process of generalisation may present the toughest problems of all.

If we now consider the main subjects of which analysis consists, our point of view becomes considerably modified. The theory of functions of a complex variable has been more adequately represented in France, Germany and Finland than in England, where the geometrical point of view has not been sufficiently developed. The theory is a sharp and useful tool for solving problems in other branches of mathematics, and few mathematicians can afford to ignore it completely; it is also a vast field for investigation of problems within its own boundaries. Within the theory itself there have been, and still are, many hard, sharp problems; but the usual procedure has been to build up by extending and generalising known results such as Picard's theorem until the whole has developed into a magnificent structure. The only drawback is that both statements and proofs tend to become wearisome in length, and even the meaning of some of the results is difficult to comprehend. Though much of the work on integral functions and Taylor series is, I think, necessarily a matter of calculation, we cannot progress far without using conformal representation; and then the geometrical ideas implied by it call for the use of topological methods. Perhaps new methods will lead to some fundamental simplification which will make the conceptions more readily intelligible, and reduce the calculations. For, if not, a large part of the theory of functions of one or more complex variables will fade out of the public interest, just as some of the algebraic work of the last century has done.

The theory of functions of a real variable is in the process of being transformed by modern methods. Much of the classical theory has its origins in Fourier's work on the expansion of an arbitrary function in a trigonometric series; for this led to the development of all the modern theories of integration, and the study of the convergence and summability of Fourier series. The study of Fourier series seems to have approached the theory of functions of a complex variable more closely in some of its recent developments, which depend on conjugate harmonic functions; but the more general theories of integration stimulated the study of sets of points and the foundations of mathematics. At the same time, the calculus of variations and the integral equations arising in mathematical physics gave rise to functional analysis; that is to say, the study of functions which depend, not on one or more variables, but on other functions. It is difficult for one who is not an expert in these branches to trace the history of development accurately; but a new point of view, more abstract, more general, and simpler in details certainly came into being. It had its origins in Fréchet's work on abstract spaces and Volterra's work on functionals, and has been more popular abroad, especially in Poland and America, than in England. General orthogonal series are considered instead of the special types of orthogonal series such as Fourier series; and the sets of points are replaced by sets of elements of abstract spaces in which only certain of the more elementary geometrical notions concerning distance, such as neighbourhood, limit point, still hold.

In this general and sometimes very abstract work the amount of calculation is comparatively small, and the methods are attractive; but it may be asked whether it actually does the job of solving definite problems as economically as the classical analysis. The answer is by no means easy to ascertain. For the results have to be translated from one form to another, and the mathematician who is familiar with abstract work seldom has the command of detail required for the transition, and vice versa. So that although for instance a classical theorem, such as Littlewood's Tauberian theorem, may be deduced from Wiener's general Tauberian theorem, the calculations required to derive it may be far from obvious, though in this particular case the connection has now been thoroughly worked out.

I have only discussed a few main trends in analysis, and those very inadequately. No single person could properly digest the mass of work turned out in recent years; the rate at which original work was produced was, I think, particularly high from 1926 to 1936. Since about 1936 there has been a much needed increase in the number of text books published. The only way to begin mathematical research, and to learn what mathematics is, is to try to solve problems; and fresh interesting problems keep on turning up in branches of mathematics which seem at first sight to be exhausted; but I think that one of the most urgent problems in analysis is that of digesting, simplifying and co-ordinating the work already done so that the essentials can be more easily assimilated.

Some Missing-Figure Divisions

By The President

For some years I have been intrigued by the type of problem called "the missing-figures problem." There are many variations, involving addition, subtraction, multiplication, division or even the extraction of square roots, but in this short article I will confine myself to division problems. Among the best known of these are "the solitary 7" and "the numberless decimal division," which are to be found in one of Dudeney's Puzzle Books.* Some others of a rather more complicated type can be found in old issues of *The Mathematical Gazette*.† The problems III and IV below are given there with their complete solutions, and problems II and VI are proposed. In the cases of problems I, II, IV, V and VI, the solutions are unique, but problem III has four solutions, a fact which makes it heavier and more difficult than the others.

In all the problems, each . indicates a missing digit 0, 1, 2, 3, 4,

* H. E. Dudeney.
† W. E. H. Berwick, *Mathematical Gazette*, Dec. 1921, Jan. 1922.

5, 6, 7, 8, 9; the digit given may occur again in the problem in places other than those given. All the processes involved are strictly correct in the usual methods of long division, e.g. there if no line of zeros, and no line begins with a zero. The values of the missing digits can be found by logical deduction, combined, is desired (though in general this is unnecessary) with judicious trial-and-error.

I. Four-threes.

1. ...)..3.3(..
2. .3.
3. ....
4. ..3.

II. Three-threes.

1. ...)..3..(..
2. .3.
3. ....
4. ..3.

III. Four-fours (four solutions)

1. ...)......4(.4..
2. ...
3. ..4.
4. ....
5. ....
6. .4.
7. ....
8. ....

IV. Five-fives.

1. ....).55..5.(.5.
2. ..5..
3. .....
4. .....
5. ....
6. ....

V. Four-sixes.

1. ...)......(.6.
2. ...6
3. ..6
4. .6.
5. ....
6. ....

VI. Seven-sevens.

1. ....7.)..7.......(..7..
2. ......
3. .....7.
4. ....
5. .7....
6. .7....
7. ....
8. ....7..
9. ......
10. ......

As is easily seen, I is an easier variation of II, of which I have not yet found a really satisfactory method of solution. Complete solutions to all the other problems are available, and those to I, V and VI are given below.

In addition to solving these problems, their formulation is of considerable interest, and there appears to be no limit, except human patience, to the number or complexity of them which can be obtained. I have not yet seen any problems in which the given figures are in fixed combination of two or three, e.g. five-seventeen and nine-thirteen problems, but these are also possible.

Solution to I.—It is obvious by inspection that the first figure of the dividend is 1, and that the last figure of each of lines three and four is 3. This latter fact requires one of the following:—

(*a*) The divisor ends in 3 and the quotient in 1,
(*b*) ,, ,, ,, 1 ,, ,, ,, 3,
(*c*) ,, ,, ,, 7 ,, ,, ,, 9,
(*d*) ,, ,, ,, 9 ,, ,, ,, 7.

(*a*) is impossible since the divisor has only three figures whilst line four has four.

(*b*) is impossible since the last two figures of the divisor are then 11 and so the first figure of the quotient must be 3 which makes lines two and four equal.

(*c*) If this be true the divisor must end in 37 and the quotient is then 19, the first figure of the divisor being 5, 6, 7, 8 or 9.

Now from lines one and two we find that line three (and so line four) is .033 or .933, neither of which is possible with the above figures for the divisor. We are thus left with (*d*), viz. divisor ..9 and quotient .7; lines two and four then give us the divisor as .19 and the quotient as 27, and line two finally gives 419 as the only possible divisor.

Solution to V.—Line four gives divisor $\times 6$ having three figures while lines two and six have each four figures:

$$\therefore\ 166 > \text{divisor} > 111,$$

and the first and last figures of the divisor are greater than 6.

Also from line four, as the second figure is 6, the divisor begins with 16, 14, 12 or 11; and in fact deeper considerations give the divisor as 161, 144, 128, or 127. Line two now gives:

(*a*) Divisor = 127, and first figure of quotient = 8, or
(*b*) ,, = 144, ,, ,, ,, ,, = 9.

We find without using complete trial and error that 144 with 969, 968 or 967 requires 3, 9 or 4 respectively in line three in place of the given 6, and 127 with 868 also requires a 3 in this place. 127 with 869 alone, in fact, gives the required solution.

Solution to VI.—Multiplying the divisor by 7 gives six figures in the product (see line six), while in two cases seven figures are given in the product (lines four and eight),

$\therefore$ (*a*) the divisor must begin with 11, 12, 13, or 14, and

(*b*) the second and fourth quotient are 8 or 9.

Now also the divisor $\times 7$ has its second figure 7, and we find (using (*a*)), that

(*c*) the divisor must begin with 111, 124, 125, 138, or 139.

The remainder of line seven must begin with 10 and therefore so must line eight. Hence from (*b*) and (*c*) we have:—

(*d*) *either* the divisor begins with 111 and the fourth figure of the quotient is 9,

or the divisor begins with 125 and the fourth figure of the quotient is 8.

If the fourth figure is 9, then as the third figure from the right in the product of line 8 is 7, the divisor must be 11197..; but this gives the divisor $\times 7$ with its second figure 8, which is contrary to the data.

$\therefore$ using (*d*), the fourth figure must be 8, and the divisor begins with 125. In fact, because of the 7 in line eight we have that the divisor is 12547 and the fourth figure of the quotient is 8. This in turn gives divisor $\times 7$ (in line six) as 878... and so line five cannot be greater than 979... (since line seven begins with 10). The remainder (line seven), therefore, begins with 101 or 100. However, line eight (= divisor $\times 8$) begins 100 and line nine has six figures.

$\therefore$ line seven begins 101.

This gives the first figure of line nine as 1 and so the last figure of the quotient is 1.

Now considering line eight again we obtain the last figure of the divisor as 1, 2, 3, or 4, and trying all four of these we find that only 3 is possible.

The solution is thus completely determined.

Answers.

I & II. 419)11313(27.

III. 846)1200474(1419; 848)1202464(1419;

943(1337174(1418; 949)1343784(1416.

IV. 3926)2559752(652.

V. 127)110363(869.

VI. 125473)7375428413(58781.

N.U.S. Congress, 1941

DURING the Easter vacation, over 1100 students from the universities of England, Scotland and Wales met in Cambridge to discuss "The Student, His Subject and Society." This Congress, nearly double the size of the previous one, is a significant indication of the increasing interest shown by students in social problems.

At the opening session, the Vice-Chancellor welcomed the Congress to Cambridge, and Dr. Stead, in his address, suggested that to value any institution, and education in particular, one must first consider the function for which it was intended, and whether it was fulfilling that function. The different commissions on science, medicine, social science, education, arts, engineering and theology then met six times to discuss the connection of their own individual subject with society and the teaching of that subject. After thorough discussion of these questions, the last meeting of each commission prepared, and in all cases passed almost unanimously, resolutions summarising the opinions expressed in the previous meetings. Some of these resolutions were put before the final session for its approval.

Of primary interest to mathematicians was the science commission. Here the first important point reached was the necessity for breaking down the rigid barriers between each branch of science and relating all to contemporary society. The academic distinctions too often drawn between the many branches, clearly hindered co-operation between them, and hence, the fullest development of each. Since student and teacher alike form part of the community in which they live, through them the existing form of society has an effect on their subject, which cannot be ignored. Moreover the progress of science has been fundamentally determined by the problems of the community, and any historical account of science must take this into account. In the case of mathematics, Professor Levy pointed out that its relation to society was mainly through the more practical sciences, which were in turn directly concerned with the problems presented to it by the particular form of society, in which they existed.

This led to a consideration of the mutual reactions of present society and science, and all agreed that capitalist society was now a hindrance to science and the community. The proofs were only too numerous—the narrow scope of, and opportunities for education, the lack of funds for research on important social problems, the misapplication of science, the suppression of inventions, and so on. The way forward for science lay in establishing a society in which the welfare of the community replaced the profit motive. This particular point was brought out in every commission and received unanimous approval.

In the discussion on the present war-time conditions as they affected scientists, it was felt that the present policy of neglecting anything not of immediate military importance was short-sighted, as science played a vital part in many war-time problems such as diet and air-raid protection. Also the tendency to shorten courses and lower standards was deplored, as was the enticement of scientists into specialised war-time activities, leaving them with few qualifications for post-war employment.

The two commissions on the teaching of science and graduate employment again stressed the necessity for linking academic learning with social practice, and produced a number of specific proposals for improvements in both fields. These, as well as the more general conclusions, were embodied in the final resolution which, carried with only one dissentient vote in about 150, was put before the final session of the whole Congress.

While any practical results can only be achieved by the whole body of students and not just by that fraction which came to Cambridge, yet this Congress, the largest and most representative student gathering ever held, after five days of intensive discussion, did suggest very clearly a way out of many of the numerous problems confronting students to-day. The official report, far more detailed and more accurate than these brief impressions, which are mainly of one commission, demands the greatest attention from all who claim to be students and not merely undergraduates.

R. F. Braybrook

A Simple Acrostic

THE year that he of apple fame
Connected with a herb became,
And from the year they called him Sir,
Both the uprights you'll infer.

A palindrome number that's also the square
Of the product of only two primes, I declare.

I am a prime; take one from me
And then a power of two there'll be.

Turn Buchan's steps and down them run
A million times to reach the sun.

Here is the clue; mix middle C.
Then if you're lucky, you'll get me.

Book Reviews

Number, the Language of Science. (2nd edition.) By TOBIAS DANTZIG, Ph.D. (George Allen & Unwin.) 10s.

Dr. Dantzig presents a book which, though primarily written for the mathematically-minded layman, will also prove useful as an introductory to pure analysis. The theory of numbers is presented from an original stand-point—its historical development. This has enabled the author to dispense with many theorems and many more tedious proofs and to concentrate on making the subject attractive. The book commends itself to the more advanced mathematician, also, as it shows his subject in an original and interesting perspective.

Three introductory chapters trace the origins of the concept of number and the evolution of numeral systems and positional notation. We gain an interesting picture of the Greeks, who mingled mysticism and mathematics, and the Hindus, who were more practical and knew better.

The body of the book records the slow growth of the numerical symbol and the subsequent enlargement of the domain of number. Here the usual textbook presentation follows the historical order. We see how Vieta's symbols led to a firm establishment of the theory of rational, and later of irrational numbers. An intriguing chapter on continuity as conceived in Newtonian times precedes some remarks on the early theory of fluxions and continuous functions. The concept of real numbers is then explained, first from Cantor's, and then from Dedekind's standpoint, similarities and differences being made clear. A discussion of imaginaries is followed by remarks on transfinite numbers, and finally a chapter on the philosophical bases of mathematics.

In this new edition 26 appendices are added, ranging in subject from the evaluation of continued fractions to the trisection of angles.

Throughout the book men's mistakes as well as their successes, aims and difficulties are all recorded, and the reader will realise how unlike the usual logical presentation did the actual theory of numbers arise. The author has kept his eye all the time on the lighter side of mathematics, and his historical pageant of number will make excellent light vacation reading for any weary mathematician. R. L. W.

UNIVERSITY MATHEMATICAL TEXTS. (Oliver & Boyd.) 5s. od.
Waves. By C. A. COULSON, M.A., Ph.D.

In this book, the author has set out to present in compact form the elementary theory of many different types of wave motion, which are usually associated with distinct branches of applied mathematics. All the problems considered are fundamentally the same, depending on the standard differential equation of wave motion. The more important solutions of this equation, which are used throughout the book, are investigated in an introductory chapter. Three chapters follow, dealing with the simpler wave phenomena—transverse waves on strings and in membranes and plane longitudinal waves. From the equations of irrotational motion of a perfect fluid, is built up in the next two chapters the theory of tidal and surface waves in liquids and of sound waves in a compressible fluid. Starting from Maxwell's equations in the next chapter, Dr. Coulson builds up the theory of electromagnetic waves. It is to be regretted that pressure of space has prevented the consideration of oblique reflection and refraction of other types of waves, but here we find a discussion of the phenomena in the case of plane polarised electromagnetic waves. Finally, some general considerations help to co-ordinate all wave types; after a discussion of wave packets and group velocities the author concludes with mention of Fraunhofer diffraction and retarded potential theory.

In the space of 150 pages it has been necessary to confine attention to the most essential points of the theory, but numerous examples at the end of each chapter help to suggest to the reader further extensions and applications. This volume should prove to be a useful introduction to a more detailed study of the subject, and in so doing it will achieve the author's aim in writing it. In short, we have a book which deals concisely with most of the work on Waves necessary for Part II of the Mathematical Tripos.

J. G. O.

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