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Published by The Archimedeans, the mathematics student society of the University of Cambridge

Thanks to the Betty & Gordon Moore Library, Cambridge

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EUREKA

THE ARCHIMEDEANS' JOURNAL

JANUARY, 1941

PRICE 8d.

EUREKA

THE JOURNAL OF THE ARCHIMEDEANS

(The Cambridge University Mathematical Society: Junior Branch of the University Mathematical Association.)

No. 5 JANUARY, 1941

Editorial

THE appearance of this issue of EUREKA has been possible only through the co-operation of both senior and junior members of the University and others who have spontaneously taken up their pens to write to the Editors. We would thank all those to whom we have been unable to express our appreciation individually.

It will be realised that it is becoming increasingly difficult to produce EUREKA, as is reflected in the slight increase in price, but we hope that it will appear again in May. So if you meet an unexpected result or wish to set a problem to fellow mathematicians, however trivial it may seem to you, remember that the Editors will welcome it,

> "For he that reads but mathematic rules
> Shall find conclusions that avail to work
> Wonders that pass the common sense of men."

Mathematical news from other Universities would undoubtedly be of great interest to all Cambridge readers and we should like to have more contributions also from those mathematicians who have been evacuated into Cambridge for "the duration."

In addition to problems and other articles, we should like to receive comments on EUREKA, or on any mathematical subject, which may be suitable for inclusion as "Letters to the Editor" in our next issue.

Contributions or suggestions should be addressed to the Editor of EUREKA, The Archimedeans, c/o The Mathematical Faculty Library, New Museums, Cambridge (not later than April 1st, 1941). The Business Manager will be pleased to give details of Advertisement Rates.

Archimedeans' Activities

Owing to unsettled conditions little was done to form last term's programme until the term had started. However, the usual high standard was maintained and all meetings were very well attended.

There were four evening meetings. Professor Sir Arthur Eddington spoke on "Theory versus Observation," and expressed the opinion that theories were not dependent on observations, but observations were dependent on theories. Over three hundred people, probably a record, were present. At a discussion on "Mathematics or Examinations," the reasons for reading mathematics at a University were considered. Dr. F. Hoyle addressed the third meeting on "The Evolution of the Stars," and gave a new theory on the accretion of interstellar matter by celestial bodies. Mr. M. H. A. Newman wound up the programme with a talk on "New Problems in the Foundations of Mathematics," and, as usual, held the attention of his audience throughout. Mr. Newman proved Gödel's Theorem that in a logical system of mathematics there is always a contradiction.

Messrs. C. A. B. Smith and W. T. Tutte are to be thanked for the use of their room for all three tea-time meetings. Smith, speaking on "Reverse Notation," an article on which appears elsewhere, sent his audience away with grave doubts as to the efficacy of the decimal notation. Tutte, addressing the second meeting on "The Problem of the Dictator," proved that a three-colouring existed for a cubical network and also considered singly complete circuits. Mr. D. B. Scott, unfortunately, had to postpone his talk owing to illness. However, he read his paper on "The Teaching of Mathematics," in the last week of term. This meeting provoked much informal discussion and it is almost certain that the subject will be further pursued at some later date.

If this term's meetings reach the same high standard, the society can claim to have had a most successful year.

J. L. K.

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THE BRIDGE GROUP

We are hoping to form a Bridge Group this term. A preliminary meeting is being held in Newnham College on Friday, 24th January, and we hope to arrange fortnightly drives after that. If sufficient need is felt Beginners' Instruction Classes will be held for those wishing to learn to play. Anyone interested in joining the Group can obtain full particulars from Miss A. Crawford (Girton). Announcements will also be made at Archimedeans' evening meetings.

C. S. B.

THE MUSIC GROUP

For various reasons, mostly connected with the fact that it was the beginning of a new year, it was not possible to arrange the first of the Music Group meetings until halfway through last term, and so there were only three meetings altogether. However, all of them were very successful and, except when the weather interfered, very well attended. The first was held in what must surely be one of the smallest rooms in Cambridge, and so of course attracted the greatest number of people. Four items were played, chief among them being Beethoven's 4th Piano Concerto. For the second meeting Dr. and Mrs. Jeffreys were good enough to lend us a room in their cottage, and several performers entertained us with piano solos, a violin sonata and some songs. This meeting in particular was thoroughly enjoyed, and proved a welcome change from the other two gramophone meetings, the second of which was unfortunately interfered with by rain. However, although nearly three-quarters of those who met in St. John's came either from that college or from Trinity, more than twenty people were interested enough to come and hear a Bach Orchestral Suite and Brahms' 4th Symphony.

This term, it is hoped to arrange three gramophone meetings and one recital. Details and dates will be on the Archimedeans' card, and it is hoped that all Archimedeans in any way interested in music will come along and join us.

P.S. and N.B.—Anyone who has any gramophone records which he is willing to lend for these, or future, meetings, is very welcome at any time (even tea-time) at D7, Chapel Court, St. John's College.

J. T. H.

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THE CHESS GROUP

In the middle of the Michaelmas Term, a Chess Group was started and two meetings were held at Peterhouse. Our second evening was devoted to a very successful lightning tournament. The players were first divided into four sections and the finals followed an "all-play-all" in each section.

We must apologise for being unable to say much about this term's activities. Negotiations about a suitable room for play are still in progress; it is hoped that meetings will be held in Emmanuel College. We propose to hold three or four meetings and, Mr. D. B. Scott has kindly consented to give a Simultaneous Display during the term. Another lightning tournament will also be held. All Archimedeans, whether they be University players or mere "wood-pushers," are invited to come along. Look for further details, which will be announced as soon as possible.

J. B. P.

Modern Geometry

By Prof. W. V. D. Hodge

I have frequently come across mathematicians who appear to regard geometry as something a little apart from the rest of pure mathematics, and I have observed that this view seems to be held by some contributors to Eureka. When an opinion like this is expressed, it is usually meant to refer only to synthetic geometry and algebraic geometry, more particularly the latter, and in this attempt to examine the reasons for this apparent ostracism of geometry I shall use the term geometry to denote only these two branches of the subject.

Dealing first with the geometry which comes in Part II of the Mathematical Tripos, I am convinced that, while it is not easy, geometry is no more difficult than many other subjects taught for the Tripos. This opinion is on the whole borne out by my experience as an examiner. I have found that as many candidates do well in geometry as in other subjects; but, on the other hand, probably more do really badly in the geometry paper than in any other paper. I think that I can explain this. In studying geometry it is more important to develop an understanding and appreciation of geometrical configurations than to obtain a mastery of technique. Once this "geometrical sense" has been acquired, the deduction of theorems and the solution of problems become easy matters, but without it the student is apt to flounder when confronted with an example, not knowing where to begin. It is the lack of this sense which produces the failures. I do not want to imply that "geometrical sense" is anything mysterious; it is within the reach of anyone who is fit to take the Tripos, but some are slower than others in acquiring it. Probably those who are slow feel the lack of a suitable textbook with which to supplement their lectures. The ideal textbook for these people has not yet been written, but it will undoubtedly come some day.

It is unfortunate that many mathematicians who in their undergraduate days are reasonably good at geometry allow themselves in later years to get out of the habit of thinking geometrically, with the result that they are unable to take any interest at all in recent developments in synthetic geometry. With a little effort to keep themselves in training, they could be in a position to take an intelligent interest in much that is done in synthetic geometry, but I am afraid that it is a case of the day not being long enough for everything, and one soon lets oneself get out of training.

Coming now to algebraic geometry, let me first dispose of a charge that is sometimes brought against algebraic geometers. It is sometimes said that they are in the habit of lifting results from algebra without bothering to prove them. This is simply silly. Geometers do study the algebraic basis of their subject, but in teaching it they have as much right to borrow results from algebra as, say, a lecturer on differential equations has to quote from the theory of functions of a complex variable.

The most important department of modern algebraic geometry is birational geometry, and here I am prepared to admit that there is some reason for the ignorance, if not suspicion, which the non-geometer displays towards the work that is done in this field. This work is, indeed, bound to remain a closed book to him unless he is prepared to take a great deal of trouble, and if he picks up casually any writing on the subject, I am not surprised that he finds it strange and not looking like ordinary mathematics at all. To understand this rather remarkable state of affairs, we must turn back the pages of geometrical history to the time when Brill and Noether wrote their famous memoir on the birational geometry of curves. This work issued a clear challenge to geometers to generalise it to surfaces and loci of higher dimensions. The first attempts to take up the challenge clearly showed that the task was no easy one, but in the last decade of the nineteenth century Castelnuovo and Enriques developed and generalised the notions of linear systems of sets of points on a curve contained in the Brill–Noether paper, and showed that the use of linear systems of curves on a surface opened up a wide avenue of progress. These two mathematicians, together with Severi and a large number of lesser geometers, then proceeded in the next twenty-five years to develop a theory of surfaces which is one of the most remarkable achievements in the history of geometry. In order to understand the work of the Italian geometers it is necessary first to study carefully the foundations of the theory of linear systems of curves on a surface, and this theory is rather deceptive in its appearance of simplicity. Behind a number of apparently simple results in the theory lie many implications, and unless one appreciates these implications one is apt, on reading a paper by, say, Severi, to fail to see why one statement implies another, with the result that a proof sometimes seems odd and inconclusive judged by ordinary mathematical standards. The technique of Castelnuovo is, indeed, so specialised that only experts can appreciate its use. Moreover, it has largely been attempts to present the theory in a simplified form that have led to the impression which some people have formed that the theory of algebraic surfaces is not completely rigorous.

There can be no doubt whatsoever of the overwhelming importance to geometry of the introduction of the methods of Castelnuovo and Enriques, but the resulting isolation of geometry, besides being regrettable in itself, has certain practical drawbacks. Naturally the methods of linear systems cannot do everything, and experience has shown that there are several places where geometers would welcome the help of other mathematicians, and it is unfortunate that ignorance of the geometry of surfaces prevents analysts and topologists from making use of wide fields which are ready for the application of their subjects.

Let me end, however, on a note of optimistic prophecy. I am convinced that the day will come, and is not very far distant, when much that is now closed to non-geometers will become familiar to a much wider range of mathematicians. The real reason why geometers had to invent special methods and plough a lonely furrow was that in 1890 algebra was not able to cope with the problems with which geometers were faced. But in recent years much progress has been made in algebraic theories, particularly the theory of ideals, and recent developments have made it possible to give algebraic proofs of a number of results in the geometrical theory of algebraic loci. Magnificent pioneer work has been done by van der Waerden and Zariski in founding an algebraic theory of loci, and as the theory develops it will be found to be much more tangible than the geometrical theory which seems to repel a number of mathematicians. This is not, of course, intended to imply that the work of the Italian school will become less important, but only that the results of this work will be thrown open to a wider circle. There is, indeed, much that can still be done most simply by means of linear systems, and by the generalisation to systems of equivalence, but by the use of the methods of modern algebra it will be possible to present this in a form which will have a more general appeal.

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Mathematicians in the Army

By J. G. Oldroyd

In a battery of artillery, it is found that three schools, three colleges and three faculties are represented by six Cambridge men of various ranks from major to gunner. One of them, who read a different subject from the rest, was at school with the captain and in college with the sergeant; before the war, the captain and sergeant met only at lectures and none of these three ever met the major. Of three men who read the same subject, two were at the same school and two were in the same college. The lieutenant and bombardier met for the first time at a University lecture, while the lieutenant's college friend read classics with the man whose school friend was in college with the lieutenant's friend at school. The sergeant was at school with a historian of the same college as the man who was at school with a man of the captain's college. Who was in college with the bombardier and who read mathematics?

A New Way of Writing Numbers

By C. A. B. Smith

MEN generally accept the common number system without question. They may say "It would be wiser to number by twelves in place of tens," but even if they have a belief or hope that some day we shall make the change, they have no idea that it is possible *at present* to make the trouble of number operations very much less.

Yet one way of increasing the quality of work with numbers is quite clear. It is by numbering down as well as up. When reading the time, one commonly says not "5.53" but "7 minutes to 6." In the same way as in writing 13 for "3 more than 10," one may write $1\bar{3}$ or 1Ɛ for "3 less than 10," putting a line over the number, or turning it round, to make the change into its opposite. The expansion of the system to greater numbers is simple, thus: 28617 = 3Ɩᔭ2Ɛ. In this way the only number-signs necessary are ᔭ, Ɛ, ᘔ, Ɩ, 0, 1, 2, 3, 4, 5, all quite small; but ς may be put in as it is a great help.

An important effect of this change is that the writing of numbers less than 0 is now possible in the same way as for those greater than 0. Take, for example, the number sixteen, 2ᔭ. Turning each number-sign round we get ᘔ4, four more than minus twenty, i.e. the opposite number. In the old system we have to put the sign "—" before "16" to make it clear that the opposite of 16 is in mind.

These powers of writing *all* numbers in the same way, and of quickly and simply changing a number into its opposite, have interesting effects. By them, taking away a number is turned into the addition of its opposite. Any number of additions and subtractions may be done at the same time: so money accounts will be made simpler. Lists of cosines will not have "take away differences" printed at the top. And so on—the reader may see for himself how much more free most work with numbers becomes because of this.

Again, in adding, because the separate number signs are as often less than as greater than 0, the sum of any line of numbers will be small, and often there will be nothing to take over into the next line. In approximations in the old system one has to make an addition of 1 to the last place kept in, if the number after it was greater than 5; in the new system this is not necessary.

Cauchy made great use of the system when he had much heavy work to do; and putting his words into English, "Additions, subtractions, multiplications, divisions, turning fractions into 'decimals,' and all other operations with numbers are made very much simpler by this way of writing numbers, in which, as I have said, we make no use of number-signs of value greater than 5."

This is only the very shortest account; those desiring a full discussion may see J. H. Johnston's small book, *The Reverse Notation* (numbering by twelves, not tens).

But is this the best we are able to do? After all, the learning of these new signs and their addition and multiplication will take some time—almost every group of two will have to be gone over again.

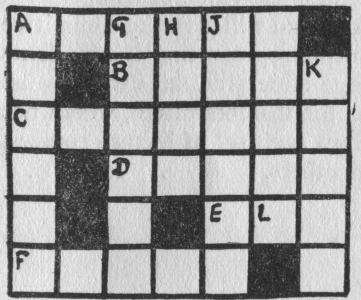
Consider a new system—numbering by sixes and changing the order of writing the separate signs. That is, we make use of the signs Ɛ, ƨ, ɪ, 0, 1, 2, 3; Ɛ is not necessary, but it is a help. We put 01 for six, 001 for six times six, or thirty-six, 1· for 1/6 and 10· for 1/thirty-six. So 21 is two and six, or eight; ƨ3, two less than three sixes, or sixteen; 3·, one half. I have not the space here to give all the reasons for making these changes, but I am hoping the reader is in agreement with me that numbering by sixes is better than numbering by tens, and my experience is that it is better even than by twelves. And in doing additions and multiplications one has to work from right to left in the old system, though the common direction of writing and reading numbers is from left to right, making addition much harder than necessary—you can test this for yourself.

The point I am attempting to make here is that the amount of learning to be done in this system is much less even than in the ten system reversed (that is, using ɪ, ƨ, and so on). One is able to make use of it in a shorter time; its use will be simpler, much quicker, and in my experience very much more free from errors—surprisingly so. Given lists of logs, sines, cosines, . . , there would be small reason for making very much use of the ten system again, for learning and using the new system is quicker, and by special lists the turning of numbers from one system to the other may be done in a moment. (And very good lists of logs, sines, and so on, have been made in the twelve system—see G. S. Terry, *Duodecimal Arithmetic*—so why not in this?)

There is only one other point—the names of the new number-signs. This is a very hard question, because the names have to be different from common words, quite separate in sound from other numbers, and yet simple. I would make the suggestion (after much testing) of *rone*, short for *reverse-one*, for ɪ; *twor*, short for *two-reverse*, for ƨ; *ther*, short for *three-reverse*, for Ɛ. These words are a bit hard and strange at first, but that feeling will go away quickly. Perhaps you will make better suggestions.

One note. This new system has its bad points, like the old one. But a frequent event is for one to see a bad point in the new system, to say, "this is not in the old," and then to see after a long time, even weeks, that in fact there was a bad point very like it in the old system. So take great care in judging it.

And now for those interested, here are some questions to be answered in the six system. They are not hard; and when they are complete, the reader putting B, the year of discovery that a system using minus number-signs was possible, against F, which is this year, will see a good reason for putting this new way into effect now.



ACROSS

A: Very near to 2π
C: $1+a^b b^c c^{a-b}$
D: d^2+1
E: Symmetric

DOWN

A: $-e$
G: William the Conqueror
H: Playing cards + cube — leap-year
J: $j!$
K: In $-1/k^2$ this comes again and again

UP

L: $l^3+m^3=q^3+r^3$ $(l, m, q, r > 0)$

(N.B.—Keep in mind that numbers with 3 may also be written with Ɛ, thus $31=$Ɛ2, the following number being increased by 1.).

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Faculty News

THE liaison committee, which forwards suggestions from undergraduates to senior members of the Faculty, has functioned satisfactorily during the past two terms. Notable results have been obtained, comparable with those achieved in previous years by the co-operation of the Society and the Research Students' Tea Club with the Faculty.

The meeting in the Easter Term discussed printed lecture notes, but as the term ended abruptly it was difficult to follow up the recommendations. Some lecturers kindly carried out the experiment with considerable success, both in Part II and Part III courses. The system may be extended, though lecture notes would be impossible in some courses. The syllabus for Part III, published last year, is available free to undergraduates on application to the Cambridge University Press.

The Committee considered the suggestion of having a room where informal discussions on mathematics could be held between and after lectures. As a result the Faculty Board have opened Room F in the Arts School from 10 a.m. to 1 p.m. each week-day for mathematical discussion among undergraduates. We hope that full and proper use will be made of this valuable facility.

Those members of last year's committee in residence for the Michaelmas Term met some senior members in December to discuss the unsatisfactory nature of supervisions in some colleges. It is thought that an improved system of inter-collegiate supervisions could be arranged and the committee is to consider a detailed scheme. Undergraduates are invited to give us their opinions on the matter—those who are satisfied as well as those who are not.

We expect that undergraduates realise the value of co-operation with the senior members of the Faculty and will give the committee their fullest support. A new committee representing the students should be elected this term. We hope undergraduates will discuss their views and forward nominations for the committee, which has four undergraduate members, one from each set of lecture courses, a research student and four senior members. H. A. E.

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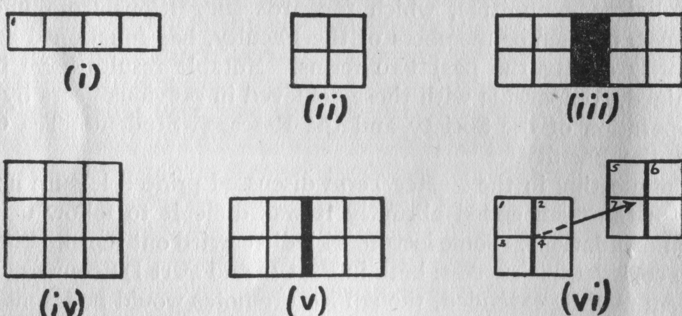
Crosswordiness

By I. J. Good

How can we measure the "crosswordiness," χ, of a crossword puzzle? It has been suggested that we should take the ratio of the number of white cells to the total number of cells. This, however, gives full marks to a puzzle with the pattern (i).



This suggests that points should not be awarded for *one-way cells* but only for *two-way cells* or *cross-word cells*. Furthermore, we should divide by the total number of white cells only, so as to give an equal mark to the patterns (ii) and (iii).

This has the advantage of making the method applicable also to patterns in which *black-outs* are replaced by *line-outs*. Observe that the pattern (v) above receives the same mark as either of the previous two ((ii) or (iii)).

But there is a serious objection to this method also, for it gives an equal mark to the two patterns (ii) and (iv).

To overcome this difficulty we may attach a different number of points to the different cross-word cells, say *the sum of the numbers of letters in the two words through the cell.*

Let the number of cells through which exactly r words pass be x_r $(r = 0,1,2)$. The number of white cells is $x_1 + x_2$, and is usually estimated most easily by subtracting the number of black-outs from the *area*, *a*, of the puzzle. For puzzles with line-outs instead of black-outs we have $x_o = 0$. Let the number of points attached to the various cross-word cells, according to the above rule, be p_i $(1 \leqslant i \leqslant x_2)$. Then, according to our definition, the cross-wordiness $\chi = (x_1 + x_2)^{-1}\sum_{i=1}^{x_2} p_i$.

Example.—A puzzle consisting of n across clues and n down clues, every across clue cutting every down clue, has

$$\chi = n(x_1 + 2n^2) \,/\, (x_1 + n^2),$$

e.g. $\chi = 2n$ for a "word-square."

Let $\nu(r)$ be the number of r-lettered words.

Let $\mu(r)$ be the number of one-way cells situated in r-lettered words. Then

$$\chi = (a - x_o)^{-1}\sum_{r=2}^{\infty}\left\{r^2\nu(r) - r\mu(r)\right\}.$$

The proof is left to the reader. This formula is usually more convenient for the calculation of χ when $\chi > 5{\cdot}5$.

$\chi = \chi(\Pi)$ is a good measure, in practice, of the average difficulty per letter of fitting words to a pattern Π, given in advance; or of the merit of a crossword puzzle composition, apart from the clues and assuming the pattern has some sort of symmetry. Puzzles with no symmetry may, however, be compared amongst themselves by the same method. I have seen an unsymmetrical puzzle with $\chi = 11{\cdot}7$, but it contained two-word solutions, abbreviations and Latin phrases. Symmetrical puzzles* with $\chi > 8$ are rare. In order that χ should be connected with the composer's difficulty it is important that this difficulty should not vary too much from one part of Π to another.

For generalised crossword puzzles in which more than two words can meet at a point, we can still define χ by adding the lengths of the words through all the junctions and dividing by the total number of white cells. For example, $\chi = 6$ in the three-dimensional crossword of figure (vi). The clues are:—

Across: 1 Adverb, 3 Preposition, 5 Exclamation, 7 Article.

Down: 1 Preposition, 2 Exclamation, 5 Exclamation, 6 Preposition.

Through: 1 Exclamation, 2 "—, si, si," 3 Q, 4 Preposition.

* I.e. symmetrical about a point or a line.

✗ ✓ # Piccadilly Underground Station

By G. A. ROBERTS

AT a given time there are on the platform, escalators and subways, and in the trains, 128 people, all of whom travel by train, and none of whom return immediately by the way they have come.

Those who have come via Leicester Square are equal in number to those who are about to travel via Leicester Square.

The number of people who arrived by Bakerloo Line is equal to the number who intend to leave by the Piccadilly Line.

The number of people who are travelling from the street to stations on the Piccadilly Line is equal to six-thirteenths of the number who change from the Piccadilly Line to the Bakerloo.

The number who arrive from Green Park and then change to the Bakerloo is equal to the number who are about to travel via Green Park.

The number who are travelling from the street to the Bakerloo is equal to four times the number who arrive in Piccadilly trains but do not use the Bakerloo Line, and of these, twice as many come from Green Park as from Leicester Square.

By how many does the number of people who use the Bakerloo Line exceed that of those who do not? (Green Park, Piccadilly, Leicester Square are successive stations on the Piccadilly Line.)

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A Theorem in Geometry

Proof due to S. N. COLLINGS

THIS is the first pure proof to be given of the theorem that *the locus of a point, such that its polars w.r.t. two conics* S, S′ *are conjugate w.r.t. the* **Φ** *conic* of* S, S′, *is the* **F** *conic† of* S, S′.

It is first proved that the locus is a conic.

Take any line l and any point P on it. This will have a unique polar p w.r.t. S. p has pole R w.r.t. Φ. r' is the polar of R w.r.t. S′. r' cuts l in Q, which has polar q' w.r.t. S′, and p, q' are conjugate w.r.t. Φ, because q' passes through R. Now P determines on l a unique Q and so P, Q are in homographic correspondence, and there are two and only two self-corresponding points. Each of these self-corresponding points has its polars w.r.t. S, S′ conjugate w.r.t. Φ, and so is on the required locus. The locus cuts any line l in two points and so is a conic. (Actually P, Q are mutually related.)

* I.e. the harmonic envelope. † I.e. the harmonic locus.

Secondly, we show that this is the F conic of S, S′.

LM is a common tangent to S, S′, touching them at **M**, **L** respectively. The polars of L w.r.t. S, S′ are MN, LM. A, B, C, D are the common points of S, S′. Consider the pencil of conics touching the tangents to S at A, B, C, D, of which Φ is a member. The pairs of tangents to the members of the pencil from M form an involution. There are two members through M whose tangents at M are the double rays of the involution; these are S and the reciprocal of S′ w.r.t. S, and so the double rays are ML, MN which separate harmonically the tangents from M to Φ, since Φ is a member of the pencil. Therefore ML, MN are conjugate w.r.t. Φ. But L lies on **F** and also on the locus. L is any one of 8 points and so **F** and the locus are identical.

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Vignère's Cipher

By M. E. Pulvermacher

A code message consists of a set of symbols which are given purely arbitrary meanings. For example, "LOFAX" might mean "I shall arrive at 12 o'clock to-morrow." A cipher is a method of writing any message so that it cannot be read immediately by anyone ignorant of that method, though many ciphers can be solved without much difficulty. The simplest ciphers are the monalphabetic substitution ciphers such as Julius Caesar's cipher or the Freemasons' Alphabet, in which each letter of the alphabet is represented by a distinct symbol. These ciphers may be solved easily by using a frequency table as E. A. Poe explains in *The Gold Bug*.

Vignère's cipher makes the frequency tables useless by using several different alphabets. Suppose the message "Where are the shells" is to be ciphered by the key word GUN which contains three letters. The 1st, 4th, 7th, . . . letters of the message are ciphered by using the alphabet in which G represents A, H represents B, and so on; the 2nd, 5th, . . . letters of the message are ciphered by the alphabet in which U, V, W, . . . represent A, B, C, . . . and the 3rd, 6th, . . . letters by the alphabet in which N, O, P. . . . represent A, B, C, . . . The message WHERE ARE THE SHELLS becomes CBRXY NXY GNY FNYYRM. Since, for example, E is represented by R or Y and Y and R can both represent L, the frequency table is useless. The weakness of this cipher lies in the fact that in a long message certain words and bigrams must recur frequently. Therefore some of these bigrams will fall repeatedly under the same letters of the key word, and will result in repeated bigrams in the cipher. Thus RE is represented twice by XY in the example above.

Obviously, if a repeated bigram in the cipher represents the same bigram of the message, the number of letters which separates the repeated bigrams must be a multiple of the number of letters in the key word. This is the basis of Karishi's method of solution in which the number of letters between any pair of repeated bigrams in the cipher is noted. The majority of these numbers will contain the same factor which equals the number of letters in the key word, say three. The letters of the cipher are then written in three columns, the 1st, 4th, 7th, . . . letters in the first column, the 2nd, 5th, . . . in the second and the remainder in the third. Each column has been ciphered by the same alphabet and can be solved by the ordinary frequency table. The Vignère cipher can be made more difficult to solve by using jumbled alphabets, but even then it can be solved by this method.

The Gronsfeld cipher, which was used by the German Army in the Franco-Prussian War, is essentially the same as Vignère's cipher and may be solved in the same way. To cipher COME HERE by the Gronsfeld method, given the key number 123, we write the key number repeatedly above the message. The letters C, E, R under 1 are moved forward one place in the alphabet, and become D, F, S; the letters O, H, E, under 2 become Q, J, G, and the letters M, E become P, H. The cipher is DQPF JHSG, which would also be obtained by Vignère's method with BCD as the key word.

The following frequency tables for the English language will be found useful in solving ciphers.

Single letters:—E; T; A, O, N, R, I, S; H; D, L, F, C, M, U; G, Y, P, W, B; V, K, X, J, Q, Z. (12–13 per cent. of the letters in a long message are Es.)

The commonest initial letters are T, A, O, S, W, I, H, C, B.

The commonest final letters are E, S, D, T, N, Y, F.

The commonest bigrams are TH, HE, AN, RE, ER, IN, ON, AT.

Here are two messages in cipher. One is a Vignère cipher and the other a substitution cipher with suppression of frequencies:—

(i)

```
VPTLR  VOGPA  XREST  KSLTX  DZMKK  WCALZ
UUDYR  ZPOPA  HRYVI  OIPWA  TKXRP  SHHII
QXAHR  VUICK  KRUET  YIGGT  ALHKJ  MPAXR
ESQBX  IGKTP  ZVFUP  UCTCA  JHPKK  MHDIE
GMSYI  ZPNDY  GVOMC  AW
```

(ii)

```
54949  19319  64915  21262  53655  27891  52531  19152
19549  41213  13941  97895  14955  39565  64116  59962
78121  31312  56871  39419  91649  16372  34995  41394
19196  49163  72876  49111  54949  55312  64781  39491
72121  35412  56871  97892  95525  31300
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Solutions of Problems

TO ALL GEOMETERS.

Geometers were asked to prove that, if E, F, G, H, J, K are arbitrary points of the edges AB, AC, AD, CD, DB, BC of a tetrahedron ABCD the four spheres α (AEFG), β (BKJE), γ (CHFK) and δ (DGJH) have a point in common.

We assume Miquel's Theorem in plane geometry which states that the circles BJK, CKH, DHJ have a common point P. Thus β, γ, δ have a common point P in the plane BCD and we define Q, R, S in a similar way in the faces CDA, DAB, ABC. α passes through Q, R, S, β through R, S, P, and so on.

Invert the whole figure with A as centre and let dashed letters denote the corresponding elements of the new figure. Q′, R′, S′ are points of the sides of the triangle E′F′G′ in the plane α'. β' is a sphere meeting α' in the circle E′R′S′, γ' meets α' in the circle F′S′Q′ and δ' meets α' in the circle G′Q′R′. By Miquel's theorem, the three circles have a point T in common. Hence α', β', γ', δ' have a point T in common and α, β, γ, δ intersect in T′. This completes the proof.

The statement that the five circumscribing spheres of the five tetrahedra formed by five planes have a point in common is false.*

PROBLEM FOR POULTRY FARMERS.

It can now be revealed that the chicken referred to in the last EUREKA was hatched out on a Friday. (Clearly this piece of information cannot now be of any interest to anyone.)

CROSSWORDS.

Across.—1: Algorithms. 9: Born. 11: Cosec. 13: Axis of zeta. 16: Cog. 17: Mu. 18: Oval. 19: A.B.C. 20: Deca. 21: T.T. 22: Hirer. 23: So. 24: Hessian. 26: Image. 29: Alto. 30: Generators. 32: NS. 33: Do. 34: Anodes.

Down.—1: Abacus. 2: Loxodromes. 3: On. 4: Rhombus. 6: To ZO. 7: Metacentre. 8: Scalar. 10: Right-hand. 12: Several. 14: State. 15: Fuchsian. 20: Di. 23: Sign. 25:Loss. 27: Geo. 28: Era. 31: To.

Across.—(*a*) 2·20462. (*f*) 366. (*h*) 8649. (*j*) 7725. (*k*) 212. (*l*) 23541. (*n*) 91189. (*q*) ·843. (*t*) 0084. (*u*) 7360. (*w*) 604. (*x*) ·577216.

Down.—(*a*) 2625. (*b*) 08. (*c*) 462. (*d*) 641. (*e*) 29·2·1944. (*f*) 3723875. (*g*) 673. (*j*) 549. (*m*) 110. (*o*) 1066. (*p*) 880. (*r*) 437. (*s*) 367. (*v*) 02 (xx rev.).

* See H. F. Baker, *Principles of Geometry*, Vol. IV, p. 10.

The Solid Angles of a Tetrahedron

By R. S. Scorer

Notation: (A) or A[BCD] denotes the solid angle between the planes ABC, ACD, ADB.

(BC) or BC[AD] denotes the solid angle between the planes BCA, BCD.

B[ACOD] denotes the solid angle subtended by the figure ACOD at B.

Let ABCD be any tetrahedron, and O any point inside it.

Construction: D^1 is on DA produced; B^1AC^1 is a straight line through A parallel to BC; AE, AF are drawn parallel to DB, DC.

Theorem 1: *(AD) + (B) + (C) = (BC) + (A) + (D).*

$(BC) = B^1C^1[BE]$,

$(B) = A[EB^1B]$, $(C) = A[FC^1C]$, $(D) = A[EFD^1]$.

$\therefore$ (AD) − (A)

$= A[D^1BC]$

= (BC) − (B) − (C) + (D), which proves the theorem.

Theorem 2: *(BC) + (CD) + (DB) + (A) − (B) − (C) − (D) = BC[OD] + CD[OB] + DB[OC] + O[BCD] − B[COD] − C[DOB] − D[BOC] = 2π, i.e. l.h.s. remains constant as A moves about in space.*

By Thm. 1 for tetrahedron OABC we have

BC[AO] + A[OBC] + O[ABC] = AO[BC] + B[AOC] + C[AOB],

and adding this to similar results for tetrahedra OACD, OADB we get

$$\{BC + CD + DB\}[OA] + (A) + O[ABC + ACD + ADB] = 4\pi + B[ACOD] + C[ADOB] + D[ABOC]$$

$\therefore$ (BC) + (CD) + (DB) − BC[OD] − CD[OB] − DB[OC] + (A) + 4π − O[BCD] = 4π + (B) + (C) + (D) − B[COD] − C(DOB) − D[BOC], which proves the theorem.

That l.h.s. of the Thm. = 2π is seen by making A descend into the plane BCD.

Theorem 3: *(AB) + (AC) + (AD) + (B) + (C) + (D) = (BC) + (CD) + (DB) + 3(A).*

By Thm. 1 (AD) + (B) + (C) = (BC) + (A) + (D)

(AB) + (C) + (D) = (CD) + (A) + (B)

(AC) + (D) + (B) = (BD) + (A) + (C).

On addition we get the required result.

Theorem 4: *(AB) + (AC) + (AD) − 2(A) = 2π.*

Adding the results of Thms. 2 and 3 we obtain

(AB) + (AC) + (AD) − 2(A)

= OB[CD] + OC[DB] + OD[BC] − 2O[BCD],

i.e., l.h.s. is independent of the position of A; and that it is equal to 2π is seen by making A descend into the plane BCD.

Applying Thm. 4 to each vertex of the tetrahedron ABCD and adding the results we get

$$\Sigma\{(AB) + (AC) + (AD) - 2(A)\} = 8\pi$$

$$\therefore 2\Sigma(AB) - 2\Sigma(A) = 8\pi, \text{ and we have}$$

THEOREM 5: $\Sigma(AB) - \Sigma(A) = 4\pi$.

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Another Mathematician's Apology

By E. CUNNINGHAM, M.A.

IT is close on thirty years now since I returned to Cambridge to teach mathematics. Professor Hardy has recently published his apology for the life of a mathematician, and I am tempted to write a little apology also. I do this as one whose interest has been mainly in teaching, which the Professor says he hates, and as one who has long since fallen out of the ranks of the promoters of new knowledge, in which the Professor excels. There is this other difference between us, that while he deals in pure mathematics in which, as has been said, you are never so happy as when you do not know what you are talking about, I have moved more in that part of mathematics which is still happily described in the headings of the papers of the Mathematical Tripos as "Natural Philosophy"; in which you must know what you are talking about, but are not always quite sure what it is that you are saying. To me the real interest has been the progressive revelation of mathematical order in the natural world. To know this gives a special sense of beauty, a feeling of awe in the presence of that which is shot through and through with the finest threads of pure reason. The origin and the justification of applied mathematics is this unfolding of pattern and harmony in the physical world. This was the stimulus to the work of Kepler, of Newton, of Maxwell, of Einstein and of all the company of their colleagues. The fact that it has given men a vast power over nature is incidental and is manifestly open to contribute as much to disaster as to welfare for the human race. The pursuit of power is never true wisdom.

There are two poets who seem to have caught the true spirit of mathematics, though neither of them got far in the study of it. In *The Prelude*, Wordsworth writes these lines:

> "Yet may we not entirely overlook
> The pleasure gathered from the rudiments
> Of geometric science. Though advanced
> In these inquiries, with regret I speak,

No farther than the threshold, there I found
Both elevation and composed delight:
With Indian awe and wonder, ignorance pleased
With its own struggles, did I meditate
On the relation those abstractions bear
To Nature's laws, and by what process led,
These immaterial agents bowed their heads
Duly to serve the mind of earth-born man;
From star to star, from kindred sphere to sphere,
From system on to system without end.

"More frequently from the same source I drew
A pleasure quiet and profound, a sense
Of permanent and universal sway,
A type, for finite natures, of the one
Supreme Existence, the surpassing life
Which—to the boundaries of space and time,
Of melancholy space and doleful time,
Superior and incapable of change
Nor touched by welterings of passion—is,
And hath the name of God."

Incidentally it has often amused me to think that Wordsworth unconsciously predicted that a hundred years later Minkowski should say of the infant study of relativity that it has brought space and time together to the melancholy and doleful position where henceforth neither of them existed in its own right but only a vague kind of combination of the two remained.

The other poet to whom I have referred is Robert Bridges, late Poet Laureate, who writes in *The Testament of Beauty*:

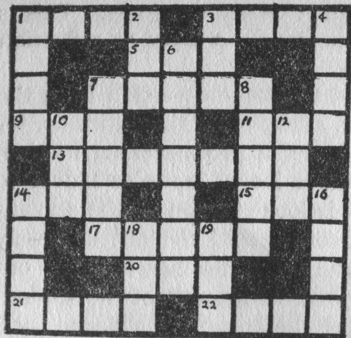
"..for since all natur is order'd (nor none will deny
that 'tis by Reason alone we are of such order aware),
all things must of their ordinance come in her court
for judgment; and 'twas thus Pythagoras coud hold
NUMBER to be the universal essence of all things:
nay, see the starry atoms in the seed-plot of heavn
stripp'd to their nakedness are nothing but Number;
and see how Mathematick rideth as a queen
cheer'd on her royal progress thru'out nature's realm;
see how physical Science, which is Reason's trade
and high profession, booketh ever and docketeth
all things in order and pattern;.."

To have been given the ability and the calling to study and to teach mathematics appears to me to be a high privilege and to carry with it a corresponding responsibility. The privilege is that of being a fellow learner and a partner in the awakening of human minds to "the music of the spheres," as Kepler called it. The responsibility is to keep the study on that level, to present our science as a queen and not as the drudge and servant of a materialistic and power-seeking society. This is one of the worst evils attendant on war; that minds which are trained, and machines which are designed, to fathom the inner secrets of the atom can be diverted to the more accurate carrying out of blowing human beings to atoms. This is nothing to do with the virtues of mathematics; it is only a reflection on the present attainment of society.

Another direction in which we find the same diversion from the pursuit of truth is to be found in the study of mathematics or, for that matter, of any other subject, under a profit-seeking motive. Far too much are our university studies looked upon as a door to a profitable career. The University is looked upon as the servant of the Appointments Board. This commercial element is closely related to the greatest difficulty which the teacher finds in our English examination system. How can the interest of the student be drawn away from the desire to get the answer to a desire for a clear understanding of the physical principles involved in a problem and an accurate expression of those principles as they bear on the problem? The study of mathematics is so wrongly divorced from the practice of speaking and writing good English. For the essence of good mathematics, like good language, is, first to know what you want to say, and then to be able to frame words which express exactly that and nothing else. The extended use of symbols should not militate against this, for symbols are only a shorthand language.

I suspect that the emphasis on accurate analysis in pure mathematics in recent years has encouraged the tendency to think of applied mathematics as a field for ingenuity and not for accurate thinking. Actually the logical difficulties in physical theory are in one sense much greater than in pure mathematics. It is required that the thinker shall not only deal logically with quantities, but also keep in mind the physical principles and from them extract the quantitative relations. Sir George Darwin once said that when a physical problem has been reduced to the dimensions of a Tripos problem it is three parts solved. That is very true and it is only a part of the greater truth that Natural Philosophy is the unravelling of the secret eternal order of the Universe. A good solution of a problem in mathematical physics always has the quality of a work of art.

And so, whether Professor Hardy hates teaching or not, it is a privilege in a place like Cambridge to be lectured to by men like him, whose delight is in eternal truth and who meditate on it day and night. In such learning we are lifted above the storms of passion, the pressure of self-will, above racial divisions. Our only pursuit is of truth and beauty; our only guarantees of success are an open mind, honesty and imagination, disciplined preparation, humility and readiness to admit mistakes. To the study of mathematics many terms apply which have commonly an ethical significance; honesty, purity, freedom from self. More of these marks of the mathematician would make a better and a more peaceful world.



A Crossword in Decimals

By E. M. White

ACROSS

1. Cube.
3. 1 *Force de cheval* = x h.p.
5. Ice.
7. $\int_{5}^{6} \frac{x^2 dx}{x-2}$.
9. This number + itself reversed = inside of 17.
11. Three roots of $x^4 - 19x^3 + 131x^2 - 389x + 420 = 0$.
13. Multiple of 22.
14. Permutation of 12.
15. Zero.
17. Can be expressed as n^4.
20. 0·1 knot.
21. L.C.M. of 18 and 14 across.
22. $\int_{0}^{\frac{1}{2}\pi} \frac{dx}{3 + 5\cos x}$.

DOWN

1. $\int_{1\cdot 5}^{2\cdot 5} \frac{dx}{x(\log_e x)^2}$.
2. Square.
3. Sum of squares of two numbers the difference of whose squares is equal to twice their sum.
4. Real root of $x^3 + 6x = 2$.
6. $2n! + 4(n+1)!$
7. Loop of $r = \sin 5\theta$ (reversed).
8. $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots$ (reversed).
10. Permutation of 11.
12. $\sum_{1}^{n} r^3 + 2\sum_{1}^{n+1} r^2 + \sum_{1}^{n+2} r$.
14. Cube upside down.
16. $n^3 + n^4 + n^5 + n^6 + n^7$.
18. Sum of first n numbers.
19. $1 - \frac{(0\cdot 5445)}{1!} + \frac{(0\cdot 5445)^3}{3!} - \frac{(0\cdot 5445)^5}{5!} + \ldots$

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Round the Table

By J. B. Parker

Seven men, Messrs. Black, Blue, Brown, Gray, Green, Purple and White sat down at a circular table laid for eight. Each man was wearing a tie and a pair of socks—and also had a car, their colours being three of the names of the other six men. There was a tie, a pair of socks and a car of each of the seven colours.

Mr. Blue sat next to the man with the green tie; between Mr. Gray and the man with white socks there sat a man wearing a white tie, and opposite him sat Mr. Green.

Mr. Brown's socks were of the same colour as the tie of the man who occupied the chair on his right, and Mr. Green wore a brown tie. The empty chair lay between Mr. Black and the man who wore a green tie. The man with black socks had a gray car, and the man with gray socks had a black car.

Mr. Purple's car was the same colour as Mr. Gray's tie, and this was of the same colour as Mr. White's socks. The man with the name of the colour of Mr. White's car wore socks of the colour of Mr. Black's car, i.e. blue. Mr. Purple's tie was of the colour of the car of the man who occupied the chair on his right, and Mr. Brown sat opposite the man with the white car. The colour of the socks of the man whose tie was the colour of Mr. Gray's car was the same as that of the tie of the man whose socks were the colour of Mr. Black's car, and this colour was not black.

Find the colours of the socks, tie, and car of each man.

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The Euclidean Spook

By D. B. Scott

Practically no important English institutions have been designed for the purposes they serve. They were intended for the quite different needs of an earlier age, and have assumed their present form, which may well be rather different from the original one, after a long process of minor changes. Our educational system is not exceptional in this respect; this is particularly noticeable in the secondary schools. Their course of instruction is not guided by any clear conception of what our society requires for the education of its citizens, but has developed gradually in a rather unsystematic manner. The geometry that is taught there is a good illustration of this, and the purpose of this article is to examine briefly how the present state of geometrical teaching came about, to what extent it justifies the claims made on its behalf, and how far it fits in with the needs of a twentieth-century democracy.

We must remember that the inspiration of our secondary schools lay in the classical studies of our old Public Schools and Grammar Schools, and it was therefore natural for the "Elements" of Euclid to form the foundation of geometrical instruction. Yet although later developments have tended to oust the classics from their dominating position, their effect on mathematical teaching has not been so marked. It is true that the modern geometrical text-book is different from those current at the beginning of the century. Some proofs have been altered to conform more with modern ideas of accuracy. The "proofs" of some theorems whose basis was merely traditional (e.g. that the sum of two adjacent angles is two right angles) are not always given, and the order of presentation has frequently been changed. But for all these superficial alterations the purpose of a school geometry course is unchanged. It is simply to give a systematic development of the proofs of the traditional theorems, and practically nothing is taught except with that end in view.

No one can deny the historical and mathematical interest and importance of the Greek geometry. But that is no guarantee of its suitability, even in its present amended form, for the average secondary pupil. We must remember that Greek mathematics was very different from our own. The modern child can do easily arithmetical problems which would have taxed the powers of the best Greek mathematicians. Algebra was not known in the world of the classics, and this tends to be reflected in their geometry (*cf.* the set of theorems on areas of rectangles which really have no place at a late stage of the geometrical course, but which should be thrown in as illustrations at a very early stage in the algebra). To the Greeks, geometry was the only subject in which reason and logic were supreme. To-day this is not the case. For instance, the logic of algebra is readily understood by children who find the logic of geometry incomprehensible, whilst the spread of the teaching of science should make it possible (even if it has not done so yet) for reasoning powers to be developed in connection with other subjects. But in spite of this, geometry is still justified as the way of teaching people to think. It is true that people with some mathematical abilities can be taught this way, though that does not mean it is the only method. But the majority of children do not learn to appreciate the notions of theoretical geometry, just because they have never been given any understanding of what reasoning means. How can they be expected to reason logically about dry matters like points, lines and angles when they have never learnt to reason even about things in which they are interested? It is like expecting children to be able to write without teaching them to read. It is really almost incredible that it should be assumed that although children have to be taught reading, writing and practically everything else, they nevertheless learn to think, which is much harder, simply by the light of nature and the examples put before them in a subject whose aims and methods most of them never properly understand.

Of course I do not propose that an attempt should be made to remedy this by teaching children formal logic or philosophy, but a great deal could be done in an informal way. Much of what is required would fit naturally into the subject known as English grammar, and ought to be taught long before any attempt is made to prove geometrical theorems. (Incidentally, one of the first things that the young school-teacher finds out is that a large proportion of his mathematical periods is spent in teaching English.) Naturally this will require new ideas of what to include in grammar lessons, but it should be possible to liven up the subject. The introduction of puzzles of a logical character, and of simple detective stories in which the class should be asked to work out, not to guess, a solution, are two possible innovations, which would not only provide a training of real value, but should commend themselves readily both to teachers and pupils. There is also a rather neglected branch of school mathematics, which can be useful in this respect, and that is co-ordinate geometry, which is usually called "Graphs" and regarded as algebra. You can teach children a lot about using their heads by discussing the deductions which can be legitimately made from a graph. This is an interesting subject, and by a suitable choice of the material presented, it should be possible to impart a great deal of useful knowledge in a palatable way, while the ability to appreciate the meaning of a graph, or any other form of statistical conjuring trick, is an essential part of the modern citizen's education.

The belief in Euclid which most educationists seem to hold is very easy to explain. Those on whom the treatment has been successful naturally endorse it, while those on whom it has failed find themselves in the same position as the courtiers confronted with the Emperor's famous new clothes. Only when the imposture is exposed can we hope to produce a school geometry in accordance with the needs of the day, for the theoretical development of Euclidean geometry takes all the available time in the normal school life. The first point we must emphasise, however, is that many of the results that are now taught are too valuable to omit, and they must be acquired, at least as experimental truths. But we must also provide the further geometrical training that contemporary society demands, and as soon as we reject the traditional justification of Euclid, it becomes reasonable to give this training priority over the theoretical geometry now taught. The war has exposed the limitations of traditional mathematical teaching, as is evidenced by the introduction of the A.T.C. in the secondary schools. I have not the time or the space to go into details about what is required. But there are one or two obvious suggestions. People need spatial perceptions, and one of the main purposes of geometrical teaching should be to develop them, and they obviously require a knowledge of solid geometry. Nor should geometry be divorced from other subjects. For instance, geography includes a great deal of geometrical matter and, in particular, map reading is a subject which could well be taught, while woodwork (or needlework) is a subject with an obvious geometrical content. It is not possible to say here how all this should be built up into a comprehensive teaching programme. It would certainly need detailed thought and careful planning. But the planning must not be, as previously, based on the idea of modifying the existing situation, but should form an independent whole based on the needs of the present and the future, rather than on the traditions of the past.

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Undergraduate Council Report

THE beginning of the Michaelmas Term found few of last year's members in residence, and under the guidance of Miss J. Margesson, the only surviving officer, the Council was reformed with Mr. Robin Lythe, of Clare, as President. Vigilance, Student Health, and Evacuee Care Committees were formed, and the term's work started.

In response to a definite desire for longer library hours on the part of many students, the University Library authorities were approached on the question of lunch-hour closing. Shortage of staff, preventing the use of a shift system, was apparently the cause, and as a result of negotiations, a rota of voluntary overseers is being prepared, and it is hoped that the reading room at least will soon be available without break throughout the day.

Despite the need of some comprehensive scheme of medical insurance felt by most students, only two colleges were found to run schemes of that nature, their small scope drastically limiting the value of their benefits. The Student Health Committee found most authorities seemed willing to consider a scheme if it could be organised, but showed no enthusiasm to help in its organisation. The Committee hopes to gain support in this matter from the Medical Society, and has amassed an impressive pile of data on the subject. It hoped to make definite proposals on the lines of a scheme that has been working successfully for some years at another university.

A series of Fora on Education was organised by the Council and became very popular. They proved a great success, breaking through the usual apathy shown here to many social problems.

Towards the end of term the constitution of the Council was reviewed. Since its founding some two years ago, its development has been so great, that many of its original over-cautious rules were now thought to be hampering the useful activity of the Council. After a lot of discussion, the Standing Committee was given basic directions for a new Constitution, to be drawn up over the Vacation and presented at the Council's first meeting in the Lent Term, 1941.

C. S. B.

Book Reviews

Introduction to the Theory of Newtonian Attraction. By A. S. RAMSEY, M.A. Pp. ix + 184. (Cambridge: at the University Press.) Price 10s. 6d. (No Index.)

Potential Theory is frequently an awkward subject to beginners, not because of its intrinsic difficulty, but because to many students it marks the break between School and University mathematics. For this reason Mr. Ramsey's book will be heartily welcomed by undergraduates, and it can be confidently recommended to them.

Nowadays, as the author points out in his preface, "the theory of potential is usually studied in connection with electricity and magnetism"; but it is usual to begin the subject with the consideration of gravitational attractions. This arrangement seems satisfactory, and the reviewer feels strongly that it is a pity to confine the study of a many-sided subject like potential theory to one particular branch, which is certainly of no greater practical or mathematical interest than any of the others. This view, however, is not generally accepted and in some Universities only the gravitational theory is taught. This procedure will almost certainly be prolonged by the provision of so useful a text-book as this, and therefore one is strongly tempted to wish that this book, in spite of its great value to students, had been written by a less expert author than Mr. Ramsey.

The text covers those parts of the subject which are required for an Honours degree, and provides an excellent introduction to harmonic functions. The purely mathematical technique of surface and volume integrals is developed *ab initio* in a very readable first chapter, but the proof of the General Principle of Convergence for volume integrals given on p. 18 appears more rhetorical than convincing. Contrary to his usual practice the author also includes an account of vector notation in the first chapter, but unfortunately the point of this is not obvious from the remainder of the text. The printing is good and there are plenty of examples taken from Cambridge and London examination papers.

D. B. S.

A Mathematician's Apology. By G. H. HARDY. Pp. 93 + vii. (Cambridge: at the University Press.) Price 3s. 6d.

At last we have a readable book giving a great mathematician's views "about mathematics" and we must be grateful to Prof. Hardy for overcoming his repugnance for writing "about mathematics" instead of writing "mathematics." After his long experience he has reached some very definite (but controversial) conclusions. He implies that "intellectual curiosity, professional pride and ambition are the dominant incentives" for research, and states emphatically "Mathematics is a young man's game"—words which are very encouraging to present-day students.

Prof. Hardy claims that real mathematics has intrinsic beauty, generality and depth in which lies its attraction for him, though he gives natural talent as the requisite for a true appreciation of mathematics. The crude utility of applied science is unattractive so that he is at variance with the appeal of "school" mathematics as presented by Hogben. The generally accepted reasons for the division between pure and applied mathematics are criticised and the difference restated; pure mathematics deals with ideas not restricted as they are in applied mathematics by the claims of physical reality. Included in the book are the article contributed to EUREKA (January, 1940) and the author's usual claim of the uselessness of mathematics.

An interesting biographical sketch concludes and partly explains the lucid though provocative essay. It is with much regret that we read the words "It is plain that my" (mathematically creative) "life is finished"; even if that were so Professor Hardy has left a shining example to the students who follow him. H. A. E.

Mr. Tomkins in Wonderland. By G. GAMOW. (Cambridge: at the University Press.) 1939. Price 7s. 6d.

THIS book is amusing, as the title suggests, but by no means childish, and indeed highly instructive. It presents in a novel way some consequences of the curvature of space, the velocity of light as the natural speed limit for material bodies and the Uncertainty Principle which are not evident in our physical world because of the particular magnitudes of the absolute constants. Furthermore, they are not explicable in the ordinary way except by the use of mathematical language, and Professor Gamow's success in illustrating the effects as he does is noteworthy.

Mr. Tomkins is taken in the first of his dreams to a world where the velocity of light is about seventy miles per hour and where the gravitational constant is a billion times larger than in our universe. The hero then observes a universe with a maximum radius one hundred miles, pulsating with a period of two hours. In other dreams he visits otherwise normal worlds where the velocity of light is ten miles per hour or where Planck's constant is sufficiently large to make a billiard ball suffer from "uncertainty." His last dream is rather fantastic, but does illustrate incidentally what is meant by the curvature of space.

Some little knowledge of modern physics is perhaps essential for a full understanding of all the subtleties of meaning, and this can be obtained from the Appendix, which consists of three popular lectures on the physical basis of the dreams. The book will be of interest to anyone who wishes to learn something of the nature of the physical world without approaching the subject mathematically.

J. G. O.

UNIVERSITY MATHEMATICAL TEXTS. (Oliver & Boyd, 4s. 6d.)
Functions of a Complex Variable. By E. G. PHILLIPS, M.A., M.Sc.

This subject is indeed suitable for a University Mathematical Text, and the author has been extremely skilful and successful in his exposition of it in a book of this size. He follows the theory from the beginning, but avoids dwelling long on the early stages with which most readers will already be familiar. He goes far enough, in the short space available, to satisfy the needs of most university students. The book is undoubtedly one of the most useful of the series. None of its matter is superfluous.

The first chapter is introductory and contains definitions and a discussion of power series and many-valued functions. The next two deal with conformal representation, and many of the most important special transformations are considered separately and summarised in a neat table. These chapters make the book extremely useful to physicists and to applied mathematicians. The following chapters give a concise exposition of the complex integral calculus and the calculus of residues. The author has presented them in a way that will not scare but rather attract the interest of those meeting for the first time these fundamental ideas. In fact the text is, in these last chapters in particular, what a University Mathematical Text should be—written for the reader and not to satisfy the author's conceit. It is well worth 4s. 6d. to any student. R. S. S.

Theory of Equations. By H. W. TURNBULL, F.R.S.

From their very nature, all the University Mathematical Texts are essentially concise little volumes of compressed information, and this particular one is no exception to the general rule. The result of this is that some of the longer and more intricate sections can only be dealt with sketchily, but in such cases Prof. Turnbull is always careful to refer the reader to a more detailed account elsewhere. This occasional enforced briefness, and also the rather noticeable scarcity of examples, give the impression that this book is of more use as a core around which, by wider reading, one can build up a knowledge of the subject, than as an actual text-book from which to work. In particular, it should be very useful to those who are looking for a short course to brush up their ideas on equations.

The book is, of course, intended for First Year Honours or Ordinary Degree students, and all that Prof. Turnbull has written (apart from two passages involving more advanced determinantal and matrical theory) can be understood by any reader with an elementary knowledge of algebra, (determinants), co-ordinate geometry and differential calculus. From this foundation he works through an introduction on polynomials and rational functions to the more well-known theory of the coefficients, roots and numerical solutions of the general equation, including Horner's and Newton's methods of solving equations, Descartes' Rule of Signs, and Sturm's Theorem. There follow chapters on particular types, such as the Binomial, the Reciprocal, the Cubic and the Biquadratic equations, with a final chapter, involving a considerable knowledge of determinants and matrices, devoted to the theory of elimination.

Such a brief survey of the contents naturally cannot hope to do full justice to all that there is in this book. It is hoped, however, that it is sufficient to give some idea of what Prof. Turnbull has managed to cover in the rather short space of 150 pages. J. T. H.

Integration of Ordinary Differential Equations. By E. L. INCE, D.Sc.

The first impression I had on opening this book was that the equations integrated were, indeed, very ordinary. The cover rashly declares that "it contains all the material needed by students of mathematics in our Universities, who do not specialise in differential equations"—but then, perhaps, the "our" refers to Scotland! By Cambridge standards, however, the book is essentially a Tripos Part I book, with some value as an introduction to the Part II course.

Bearing this in mind, it is a well presented, logically developed book. For example, where most writers seem to be content to extract integrating factors from the "blue," Dr. Ince gives a short, but illuminating, account of their properties in connection with linear first order equations, making their discovery a far less mystifying process. Again, the operator "D" is given a mathematical status, instead of being left, as it often is, an airy nothing which is called to one's aid like Cinderella's fairy godmother. The "guess and fit in" attitude to differential equations, a most bewildering antithesis of the usual method in mathematics, is very successfully excluded.

The subjects dealt with in this book are, mainly, of the type conventionally assigned to Elementary Differential Equations. I had hoped to see the Heaviside Operator explained and its value in solving simultaneous differential equations demonstrated. This hope was in vain. Instead, we march steadily through linear first order equations and their geometrical applications, plunge into the jungle of general equations of miscellaneous order and degree, emerging into the bright and homely sunlight of linear equations with constant coefficients; thence on to linear equations with certain variable coefficients and simultaneous equations. We end on the comparative peak of solutions in series, the hypergeometric, Legendre and Bessel equations being considered. The journey is made under the leadership of a very skilled guide and we miss none of the points of interest along the path.

To sum up, it is a book dealing with the usual elementary differential equations in a very efficient manner, well printed and very inexpensive. If this fits in with your requirements, I thoroughly recommend it.

B. D. B.

Integration. By R. P. Gillespie, Ph.D.

This book may be used as an accompaniment to the course in Mathematical Methods for Part II of the Mathematial Tripos or the course in Advanced Mathematics given to scientists.

Its opening chapters are based upon geometrical intuition and, by avoiding logical subtleties, help the reader to acquire that firm working knowledge of the subject which is an essential preliminary to a more rigorous discussion. After an account of the more elementary methods of integration, multiple integrals are introduced and these lead naturally to a discussion of curvilinear and surface integrals, including the important theorem of Gauss. A subsequent chapter on infinite integrals contains a treatment of Gamma and Beta functions.

The concluding chapters attempt a rigorous development of the whole subject in terms of the Riemann Integral. Here one receives the impression that little has been achieved by this procedure and that the discussion might profitably be reserved for a book confined to rigorous analysis, where it could be more complete and less incongruous. However, incidental topics such as orthogonal functions prove interesting and somewhat mitigate the concentrated severity.

This is a book which will be found useful but not inspiring.

K. R. A.

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