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Contents

1 Editorial

4 Prelude — Miranda Mowbray, Richard Pennington, Eddy

5 The Weird And Wonderful Chemistry Of Audioactive Decay — John Conway

17 The International School for Advanced Studies, Trieste — Dennis Sciama

19 The Problems Drive — Paul Smith

23 Token Cynic — Bob Dowling

25 A Storm in a Teacup — Paul Matthews

30 The Society — The Last Recorder

31 Situations Vacant — Eddy

34 Was Descartes' mathematical physics Mathematical Physics ? — Piers Bursill-Hall

45 Why 12 x 2 = 25 — Ian White

47 Modelling using the Poisson Distribution — D.J. Colwell, J.R. Gillett

50 Conference — Douglas Quadling

51 Answers to the Problems Drive — Paul Smith

54 Loose Ends

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Editorial

The Editorial policy of *Eureka* should, in my view, be designed to interest mathematics students at Cambridge. This is not to say that it should be full of mathematics; there are a great many Cambridge mathematics students who have more interests than just their work, and some who have little interest in mathematics at all. However, this policy can get nowhere without mathematics students chosing to write about whatever it is that interests them. I will not believe that there is noone who has an interest sufficiently strong that they would want to interest others in it. Yet, for the time being, I must live with the shadow of *Eureka*'s recent past and make the most of what I have managed to collect together. So I do not expect *Eureka 46* to go down as one of the Memorable Issues that history will look back to; I seek mainly to get through to readers that they can have a say, that articles other than mathematics can be a legitimate part of the journal of the mathematics faculty society; that they should be prepared to say what they want to get in return for their subscriptions to the Society. If you remain mute, noone will know what you want, so you won't get it. Speak, friend, and enter. Then perhaps *Eureka 47* or *48* may go down in history.

I would hope that at least some of those with interests other than straight mathematics will pluck up the courage to write something for future issues of *Eureka*, or for that matter to try to get their voices heard in the running of the Society, perhaps even going so far as to seek election to posts within the Society.

If they do not, we will all continue to lose out.

I have several minor items coming back in response to *Eureka* 45. The first of these is a little wad of silvery paper, on each sheet of which a message appears; on the first in Russian, on the second in French and on the third in English.

Tell you what the Russian says and I shall never show my face again, for it is all Russian to me (and even that I could not swear to), but what sense I can make of it suggests that it is similar in content to the French;

Messieurs,
Kolmogorov, le probabiliste,
ne voulait pas être automobiliste.
Après avoir mangé
son déjeuner, il expliqua que les automobiles
sont pleines de danger.

Lobatchevsky
vit Nevsky
Prospekt. En disant que les murs des maisons
n'étaient pas tout parallèles,
lui et Tchebychev avaient raison.

Agréez, messieurs, mes salutations les plus distinguées.
Blanche Descartes. (signed)

and the English;

Dear Eureka Editors,
Would you like to see some Russian Clerihews, like these ?

Kolmogorov
had no love of lorries, nor of
motors. He explained it thus:
probability calculus shows us
that it is much safer to go by bus.

Lobachevsky
saw Nevsky
Prospekt. He and Chebyshov knew well
that the house walls were not exactly parallel.
[Space is non-euclidean.]

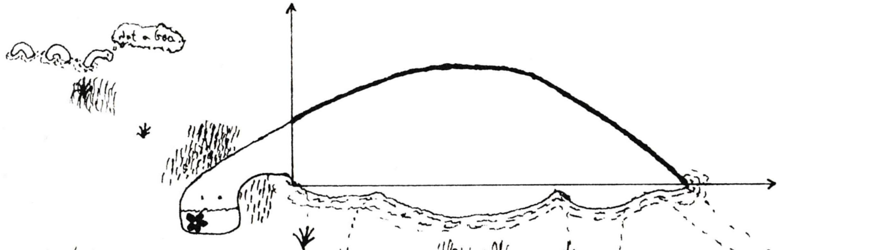
sincerely,
B. Descartes.

I also received, from the then editor of *QARCH*, Maxim Tingley, a copy of an anthem for the Archimedeans, suggested, I believe, in an earlier edition of *Eureka*.

On the subject of *QARCH*, we now have a new editor, Paul Balister. We may well see *QARCH* coming out termly, as of old, instead of only once in two years. I wish Paul the best of luck.

The third miscellaneous item is a scribbled message from Mark Goodman suggesting that the standard 'elephant inside a boa-constrictor' model of probability curves expounded in 'As

Any Brontosaurus Could Tell You, It's Not A Paradox' need not overrule the more appropriately shaped brontosaur. Brontosauri did spend large amounts of time in swamps, he points out, so the horizontal axis is the water-level, and the legs are hidden. I'm sure this controversy will blaze for years to come.



a probabilistic Brontosaurus in its natural habitat

Mark S. Adams of the Georgia Institute of Technology, in the USA, has sent us several copies of his highly colourful publication "Archimedean & Platonic Solids" (Geodesic Publications, $5.oo, 1985, -A3 paperback, c.30 pages). I cannot claim to have read it with enough care to understand, but it does have lots of pretty pictures, being done entirely in green, blue, red and yellow (in roughly that order of how much is used) on white. This produces a pleasing sensation of retinal overload. If anyone else wishes to take a look, or to read it carefully, see me (Eddy), 'though I believe the Business Manager has a few copies as well.

Finally, I close with a terminal clerihew supplied to me by John Greenlees whilst I was trying to get someone to translate the Russian. Apparently this one has something to do with category theory. I shall be amused if anyone can manage a shorter clerihew.

Kan
Can
Extend
The End.

Prelude

Miranda Mowbray, Richard Pennington and Edward Welbourne.

The editor has spent a very great deal of time obtaining Prof. Conway's article on The Elements of Audioactive Chemistry, and feels that some of you may be interested in the somewhat tortuous route whereby the problem came to Prof. Conway's attention.

The earliest appearance of the problem that we know of dates back to 1977, at the International Mathematical Olympiad in Belgrade, Yugoslavia. It is a well-known sociological phenomenon that, if one puts a sufficiently large number of mathematicians into a sufficiently small area, someone is going to start setting puzzles - and, in this case, some very interesting puzzles were indeed set, entirely independently of the official competition. The Dutch contingent perpetrated the following puzzle against the British team:

1
11
21
1211
111221
312211

What is the next line?

(Try solving it yourself! Answer in Prof. Conway's article.)

The Dutch team may have cooked this up themselves but we suspect that it is older; however, Richard was on the relevant British team, and duly brought the problem to Cambridge, where he inflicted it on Miranda, who relates that...

" When I first showed this puzzle to one of my friends, he thought for some time and then gave an agonised cry, 'I've solved it - but you need a really TWISTED mind to think of that!' I showed it to several arts students, who were all baffled, which is surprising as it requires no mathematical skills beyond counting. From my mathematical friends I got the same response as the initial one; silence and furious thinking for between two and thirty minutes followed by anguished howling. If hideous noises were heard echoing down the corridors of Newnham it was a good bet I'd asked that puzzle again."

Miranda published the puzzle in **2-Manifold**, of which Richard showed a copy to Eddy, who in turn tormented a few people. Prof. Conway lectured the Algebra III course in Michaelmas 1983 with a lecture to spare at the end, so his students invited him to a party in lecture room A at the time set aside for the spare lecture. In the course of this Eddy told the puzzle to the good Professor, who immediately started looking at the general case, rapidly noticing the few properties that had previously been seen - such as that digits other than 1,2 and 3 don't occur naturally - and then went on to guess that the strings would split; further developments came later. The following Spring he met Eddy in Sainsbury's, mentioned the discovery of the elements and promissed an article to **Eureka**. A year and a half later, here it is!

The Weird and Wonderful Chemistry of Audioactive Decay

J.H.Conway

Introduction.

Suppose we start with a string of numbers (i.e. positive integers), say

5 5 5 5 5.

We might describe this in words in the usual way as 'five fives', and write down the derived string

5 5.

This we describe as 'two fives', so it yields the next derived string

2 5

which is 'one two, one five',giving

1 2 1 5

namely 'one one, one two, one one, one five', or

1 1 1 2 1 1 1 5

and so on. What happens when an arbitrary string of positive integers is repeatedly **derived** like this?

I note that more usually one is given a sequence such as

55555 ; 55 ; 25 ; 1215 ; 11121115 ;

and asked to guess the generating rule or the next term. The history of this problem is described elsewhere in this issue of ***Eureka***.

The numbers in our strings are usually single-digit ones, so we'll call them digits and usually cram them together as we have just done. But occasionally we want to indicate the way the numbers in the string were obtained, and we can do this neatly by inserting commas recalling the commas and quotes in our verbal descriptions, thus:

```
        5 5 5 5 5
          ,5 5,
          ,2 5,
        ,1 2,1 5,
    ,1 1,1 2,1 1,1 5,
          . . .  .
```

The insertion of these commas into a string or portion thereof is called parsing.

We'll often denote repetitions by indices in the usual way, so that the derivation rule is

$$a^{\alpha}b^{\beta}c^{\gamma}d^{\delta}\cdots \rightarrow \alpha a\beta b\gamma c\delta d\cdots$$

When we do this it is always to be understood that the repetitions are collected maximally, so that we must have

$$a\neq b,\ b\neq c,\ c\neq d,\ \cdots .$$

Since what we write down is often only a <u>chunk</u> of the entire string (that is, a consecutive subsequence of its terms), we often use the square brackets "[" or "]" to indicate that the apparent left or right end really is the end. We also introduce the formal digits

0, as an index, to give an alternative way of indicating the ends (see below)
X for an arbitrary digit, possibly 0, and
$\neq n$ for any digit (maybe 0) other than n.

Thus $X^0a^{\alpha}b^{\beta}c^{\gamma}$ means the same as $[a^{\alpha}b^{\beta}c^{\gamma}$

$a^{\alpha}b^{\beta}c^{\gamma}X^0$ means the same as $a^{\alpha}b^{\beta}c^{\gamma}]$

$a^{\alpha}b^{\beta}c^{\gamma}X^{\neq 0}$ means $a^{\alpha}b^{\beta}c^{\gamma}$ followed by another digit, and

$a^{\alpha}b^{\beta}c^{\gamma}(\neq 2)^{\neq 0}$ means that this digit is not a 2.

I'm afraid that this heap of conventions makes it quite hard to check the proofs, since they cover many more cases than one naively expects. To separate these cases would make this article very long and tedious, and the reader who really wants to check all the details is advised first to spend some time practising the derivation process. Note that when we write

$$L \rightarrow L' \rightarrow L'' \rightarrow \cdots$$

we mean just that every string of type L derives to one of type L', every string of type L' derives to one of type L", and so on. So when in our proof of the Ending Theorem we have

$$n^n] \xrightarrow{(n\neq 2)} n^{\neq n}] \longrightarrow n^1]$$

the fact that the left arrow is asserted only when $n\neq 2$ does not excuse us from checking the right arrow for n=2. (But, since n>1 is enforced at that stage in the proof, we needn't check either of them for n=1.)

By applying the derivation process n times to a string L we obtain what we call its <u>n'th descendant</u>, L_n. The string itself is counted among its descendants, as the 0^{th}.

Sometimes a string factors as the product LR of two strings L and R whose descendants never interfere with each other, in the sense that $(LR)_n = L_nR_n$ for all n. In this case, we say that LR <u>splits</u> as L.R (dots in strings will always have this meaning). It is plain that this happens just when (L or R is empty or) the last digit of L_n always differs from the first one of R_n. Can you find a simple criterion for this to happen? (When you give up, you'll find the answer in our Splitting Theorem.)

Obviously, we call a string with no non-trivial splittings an <u>atom</u>, or **<u>element</u>**. Then every string is the split product, or **<u>compound</u>**, of a certain number of elements, which we call the elements it <u>involves</u>. There are infinitely many distinct elements, but most of them only arise from specially chosen starting strings. However, there are some very interesting elements that are involved in the descendants of **every** string except the boring ones [] and [22]. Can you guess how many of them there are? (Hint: we have given them the names

Hydrogen, Helium, Lithium, ··· , Uranium.)

It's also true (but **ASTONISHINGLY** hard to prove) that **<u>every</u>** string eventually decays into a compound of these elements, together with perhaps a few others (namely isotopes of **Plutonium** and **Neptunium**). Moreover, all strings except the two boring ones increase in length exponentially at the same constant rate. (This rate is roughly 1.30357726903: it can be precisely defined as the largest root of a certain algebraic equation of degree 71.) Also, the relative abundances of the elements settle down to fixed numbers (zero for Neptunium and Plutonium). Thus, of every million atoms about 91790 on average will be of Hydrogen, the commonest element, while about 27 will be of Arsenic, the rarest one.

You should get to know the common elements, as enumerated in our Periodic Table (pages 8 and 9). The abundance (in atoms per million) is given first, followed by the atomic number and symbol as in ordinary chemistry. The actual digit-string defining the element is the numerical part of the remainder of the entry, which, when read in full, gives the derivate of the element of next highest atomic number, split into atoms. Thus, for example, the last line of the Periodic Table tells us that Hydrogen (*H*) is our name for the digit-string 22, and that the next higher element, Helium (*He*), derives to the compound

Hf.Pa.H.Ca.Li

which we might call

"Hafnium-Protactinium-Hydrogen-Calcium Lithide"!

Not everything is in the Periodic Table! For instance, the single digit string "1" isn't. But watch:

1
11
21
1211
111221
312211
13112221
11132.13211 = *Hf.Sn*

after a few moves it has become Hafnium Stannide! This is an instance of our "Cosmological Theorem", which asserts that the exotic elements (such as "1"), all disappear soon after the Big Bang.

<u>The Periodic Table.</u> (Uranium to Silver)

abundance:-	n	E_n	E_n inside the derivate of E_{n+1} :
102.56285249	92	*U*	3
9883.5986392	91	*Pa*	13
7581.9047125	90	*Th*	1113
6926.9352045	89	*Ac*	3113
5313.7894999	88	*Ra*	132113
4076.3134078	87	*Fr*	1113122113
3127.0209328	86	*Rn*	311311222113
2398.7998311	85	*At*	*Ho*.1322113
1840.1669683	84	*Po*	1113222113
1411.6286100	83	*Bi*	3113322113
1082.8883285	82	*Pb*	*Pm*.123222113
830.70513293	81	*Tl*	111213322113
637.25039755	80	*Hg*	31121123222113
488.84742982	79	*Au*	132112211213322113
375.00456738	78	*Pt*	111312212221121123222113
287.67344775	77	*Ir*	3113112211322112211213322113
220.68001229	76	*Os*	1321132122211322212221121123222113
169.28801808	75	*Re*	111312211312113221133211322112211213322113
315.56655252	74	*W*	*Ge*.*Ca*.312211322212221121123222113
242.07736666	73	*Ta*	13112221133211322112211213322113
2669.0970363	72	*Hf*	11132.*Pa*.*H*.*Ca*.*W*
2047.5173200	71	*Lu*	311312
1570.6911808	70	*Yb*	1321131112
1204.9083841	69	*Tm*	11131221133112
1098.5955997	68	*Er*	311311222.*Ca*.*Co*
47987.529438	67	*Ho*	1321132.*Pm*
36812.186418	66	*Dy*	111312211312
28239.358949	65	*Tb*	3113112221131112
21662.972821	64	*Gd*	*Ho*.13221133112
20085.668709	63	*Eu*	1113222.*Ca*.*Co*
15408.115182	62	*Sm*	311332
29820.456167	61	*Pm*	132.*Ca*.*Zn*
22875.863883	60	*Nd*	111312
17548.529287	59	*Pr*	31131112
13461.825166	58	*Ce*	1321133112
10326.833312	57	*La*	11131.*H*.*Ca*.*Co*
7921.9188284	56	*Ba*	311311
6077.0611889	55	*Cs*	13211321
4661.8342720	54	*Xe*	11131221131211
3576.1856107	53	*I*	311311222113111221
2743.3629718	52	*Te*	*Ho*.13221133122211
2104.4881933	51	*Sb*	*Eu*.*Ca*.3112221
1614.3946687	50	*Sn*	*Pm*.13211
1238.4341972	49	*In*	11131221
950.02745646	48	*Cd*	3113112211
728.78492056	47	*Ag*	132113212221

The Periodic Table. (Palladium to Hydrogen)

abundance:-	n	E_n	E_n inside the derivate of E_{n+1} :
559.06537946	46	*Pd*	111312211312113211
428.87015041	45	*Rh*	311311222113111221131221
328.99480576	44	*Ru*	*Ho*.132211331222113112211
386.07704943	43	*Tc*	*Eu*.*Ca*.311322113212221
296.16736852	42	*Mo*	13211322211312113211
227.19586752	41	*Nb*	1113122113322113111221131221
174.28645997	40	*Zr*	*Er*.12322211331222113112211
133.69860315	39	*Y*	1112133.*H*.*Ca*.*Tc*
102.56285249	38	*Sr*	3112112.*U*
78.678000089	37	*Rb*	1321122112
60.355455682	36	*Kr*	11131221222112
46.299868152	35	*Br*	3113112211322112
35.517547944	34	*Se*	13111321222113222112
27.246216076	33	*As*	11131221131211322113322112
1887.4372276	32	*Ge*	31131122211311122113222.*Na*
1447.8905642	31	*Ga*	*Ho*.13221133122211331
23571.391336	30	*Zn*	*Eu*.*Ca*.*Ac*.*H*.*Ca*.312
18082.082203	29	*Cu*	131112
13871.124200	28	*Ni*	11133112
45645.877256	27	*Co*	*Zn*.32112
35015.858546	26	*Fe*	13122112
26861.360180	25	*Mn*	111311222112
20605.882611	24	*Cr*	31132.*Si*
15807.181592	23	*V*	13211312
12126.002783	22	*Ti*	11131221131112
9302.0974443	21	*Sc*	3113112221133112
56072.543129	20	*Ca*	*Ho*.*Pa*.*H*.12.*Co*
43014.360913	19	*K*	1112
32997.170122	18	*Ar*	3112
25312.784218	17	*Cl*	132112
19417.939250	16	*S*	1113122112
14895.886658	15	*P*	311311222112
32032.812960	14	*Si*	*Ho*.1322112
24573.006696	13	*Al*	1113222112
18850.441228	12	*Mg*	3113322112
14481.448773	11	*Na*	*Pm*.123222112
11109.006821	10	*Ne*	111213322112
8521.9396539	9	*F*	31121123222112
6537.3490750	8	*O*	132112211213322112
5014.9302464	7	*N*	111312212221121123222112
3847.0525419	6	*C*	3113112211322112211213322112
2951.1503716	5	*B*	1321132122211322212221121123222112
2263.8860325	4	*Be*	111312211312113221133211322112211213322112
4220.0665982	3	*Li*	*Ge*.*Ca*.312211322212221121123222112
3237.2968588	2	*He*	13112221133211322112211213322112
91790.383216	1	*H*	*Hf*.*Pa*.22.*Ca*.*Li*

The Theory.

We start with some easy theorems that restrict the possible strings after the first few moves. Any chunk of a string that has lasted at least n moves will be called an n-day-old string.

The One-Day Theorem. Chunks of types

$$,a\ x,b\ x, \qquad x^{4 \text{ or more}} \quad \text{and } x^3y^3$$

don't happen in day-old lists. (Note that the first one has a given parsing.)

Proof. The first possibility comes from x^ax^b, which, however, should have been written x^{a+b}, in the previous day's string. The other two, however parsed, imply cases of the first.

The Two-Day Theorem. No digit 4 or more can be born on or after the second day. Also, a chunk 3x3 (in particular 3^3) can't appear in any 2-day-old list.

Proof. The first possibility comes from a chunk x^{4} or more, while the second, which we now know must parse ,3 x,3 y, can only come from a chunk x^3y^3, of the previous day's string.

When tracking particular strings later, we'll use these facts without explicit mention.

The Starting Theorem. Let R be any chunk of a 2-day-old string, considered as a string in its own right. Then the starts of its descendants ultimately cycle in one of the ways

$$[\,] \quad \text{or} \quad [1^1X^1 \to [1^3 \to [3^1X^{\neq 3} \quad \text{or}$$

$$[2^2] \quad \text{or} \quad [2^21^1X^1 \to [2^21^3 \to [2^23^1X^{\neq 3}$$

If R is not already in such a cycle, at least three distinct digits appear as initial digits of its descendants.

Proof. If R is non-empty and doesn't start with 2^2, then it

either starts with a 1 and is of one of the types

$[1^1X^{0 \text{ or } 1}$ or $[1^1(2^{2 \text{ or } 3}$ or $3^2)$ or $[1^2X^{1 \text{ or } \neq 1}$ or $[1^3$

or starts with a 2 and is of one of the types

$[2^1X^{2 \text{ or } \neq 2}$ or $[2^3$

or starts with a 3 and is of one of the types

$[3^1X^{3 \text{ or } \neq 3}$ or $[3^2X^{3 \text{ or } \neq 3}$

or starts with some n>3 and has form $[n^1$.

It is therefore visible in

$$\begin{tikzcd}[column sep=small]
& {[}1^1 2^3 \arrow[dr] & {[}3^2X^3 \arrow[dr] & & {[}1^1] \arrow[dr] & {[}3^2X^{\neq 3} \arrow[dr] & {[}n^1 \arrow[dr] & & {[}2^3 \arrow[dr] & \\
{[}3^1X^3 \arrow[r] & {[}1^1 3^2 \arrow[r] & {[}1^2X^1 \arrow[r] & {[}2^1X^2 \arrow[r] & {[}1^1 2^2 \arrow[r] & {[}1^2X^{\neq 1} \arrow[r] & {[}2^1X^{\neq 2} \arrow[r] & {[}1^1X^1 \arrow[r] & {[}1^3 \arrow[r] & {[}3^1X^{\neq 3} \arrow[ll, bend left]
\end{tikzcd}$$

which establishes the desired results for it.

This proves the theorem except for strings of type $[2^2R'$ all of whose descendants start with 2^2. This happens only if no descendant of R' starts with a 2, and so we can complete the proof by applying the results we've just found to R'.

__The Splitting Theorem__. A 2-day-old string LR splits as L.R just if one of L and R is empty or L and R are of the types shown in one of:

L	R	
$n]$	$[m$	$(n\geqslant 4, m\leqslant 3)$
$2]$	$[1^1X^1$ or $[1^3$ or $[3^1X^{\neq 3}$ or $[n^1$	$(n\geqslant 4)$
$\neq 2]$	$[2^2 1^1 X^1$ or $[2^2 1^3$ or $[2^2 3^1 X^{\neq 3}$ or $[2^2 n^{(0 \text{ or } 1)}$	$(n\geqslant 4)$

Proof. This follows immediately from the Starting Theorem applied to R and the obvious fact that the last digit of L is constant.

Now we investigate the evolution of the end of the string!

__The Ending Theorem__. The end of a string ultimately cycles in one of the ways:

$$\begin{tikzcd}
2.311322113212221] \arrow[r] & 2.13211322211312113211] \arrow[d] \\
2.12322211331222113113112211] \arrow[u] & 2.1113122113322113111221131221] \arrow[l]
\end{tikzcd}$$

$$\begin{tikzcd}
2.31221132221222112112322211n] \arrow[d, shift left] \\
2.1311222113321132211221121332211n] \arrow[u, shift left]
\end{tikzcd} \qquad (n>1)$$

or
$$\begin{tikzcd} 2^2] \arrow[loop above] \end{tikzcd}$$

[Note: our splitting theorem shows that these strings actually do split at the dots, although we don't use this.]

Proof. A string with last digit 1 must end in one of the ways visible in

$1^{\geqslant 3}] \to (\neq 2)^X 1^1] \to (\neq 2)^X 1^2] \to 2^1$ or $^{\geqslant 4}1^1] \to 2^1$ or $^{\geqslant 3}1^2] \to 2^2 1^1] \to 2^2 1^2] \to 2^3 1^1]$

and its subsequent evolution is followed on the right hand side of figure 1.

A string with last digit n>1 must end $n^n]$ or $n^{\neq n}]$ and so evolves via

$$\overset{(n=2)}{\circlearrowleft}\; n^n] \xrightarrow{(n\neq 2)} n^{\neq n}] \to n^1] \to 1n] \to 11n] \to (\neq 1)11n] \to 211n] \to 2211n]$$

and the last string here is the first or second on the left of Figure 1.

```
(≠2)2211n]    (n>1)            (≠2)2221]
(≠2)22211n]                        3211]
    32211n]                        31221]
    322211n]                       3112211]
(≠3)332211n]                       3212221]
    2322211n]                      312113211]
    21332211n]                     3111221131221]
    211232221ln]               (≠3)33122211311221l]
    22112133221ln]                 2.311322113212221]
    221121123222ln]                2.132113222113121132ll]
    22112211213322lln]             2.11131221133221131112211312 21]
    221222112112322211n]           2.311311222.1232221133122211311221l]
    2113221122112133221ln]         2.1112133.22.12.311322113212221]
    22113222122211211232221ln]
    221133211322112211213322lln]                    (period 4)
    22.12.312211322212221121123222lln]
    2.1311222113321132211221121332211n]              (period 2)
    2.11132.13.22.12.31221132221222112112322211n]
```

Figure 1. The evolution of endings other than $2^2]$.

This figure proves the theorem except for the trivial case $2^2]$. (When any of these strings contains a dot, its subsequent development is only followed from the digit just prior to the rightmost dot.)

We are now ready for our first major result:

The Chemical Theorem.

a) The descendants of any of the 92 elements in our Periodic Table are compounds of those elements.

b) All sufficiently late descendants of any of these elements other than Hydrogen involve all of the 92 elements simultaneously.

c) The descendants of any string other than [] or [22] also ultimately involve all of those 92 elements simultaneously.

d) These 92 elements are precisely the common elements as defined in the introduction.

Proof. a) follows instantly from the form in which we have presented the Periodic Table.

b) It also follows that if the element E_n of atomic number n appears at some time t, then for any m<n, all of the elements on the E_m line of the table will appear at the later time t+n-m. In particular,

E_n at t $\Longrightarrow$ $Hf\&Li$ at t+n-1 (if n≥2),
$Hf\&Li$ at t $\Longrightarrow$ $Hf\&Li$ at t+2 & t+71,
Hf at t $\Longrightarrow$ $Sr\&U$ at t+72-38,
U at t $\Longrightarrow$ E_n at t+92-n.

From these we successively deduce that if any of these 92 elements other than Hydrogen is involved at some time t_0, Hafnium and Lithium will simultaneously be involved at <u>some</u> strictly later time $< t_0+100$, and then both will exist at <u>all</u> times $\geq t_0+200$, Uranium at all times $\geq t_0+300$, and <u>every</u> other one of these 92 elements at all times $\geq t_0+400$.

In other words, once you can fool some of the elements into appearing some of the time, then soon you'll fool some of them all of the time, and ultimately you'll be fooling all of the elements all of the time!

c) If L is not of form $L'2^2]$, this now follows from the observation that Calcium (digit-string 12) is a descendant of L, since it appears in both the bottom lines of Figure 1. Otherwise we can replace L by L', which does not end in a 2.

d) follows from a),b),c) and the definition of the common elements.

Now we'll call an arbitrary string <u>common</u> just if it's a compound of common atoms.

<u>The Arithmetical Theorem</u>.

a) The lengths of all common strings other than boring old [] and [22] increase exponentially at the same rate $\lambda>1$.
b) The relative abundances of the elements in such strings tend to certain fixed values, all strictly positive.

<u>Notes</u>. Since each common element has at least 1 and at most 42 digits we can afford to measure the lengths by either digits or atoms - we prefer to use atoms. The numerical value of λ is 1.30357726903; the abundances are tabulated in the Periodic Table.

<u>Proof</u>. Let v be the 92-component vector whose (i)-entry is the number of atoms of atomic number i in some such string. Then at each derivation step, $\mathbf{v}$ is multiplied by the matrix $\mathbf{M}$ whose (i,j)-entry is the number of times E_j is involved in the derivate of E_i. Now our Chemical Theorem shows that some power of $\mathbf{M}$ has strictly positive (i,j)-entries for all $i\neq 1$ (the (1,j)-entry will be 0 for $j\neq 1$, 1 for $j=1$, since every descendant of a single atom of Hydrogen is another such).

Let λ be an eigenvalue of $\mathbf{M}$ with the largest possible modulus, and $\mathbf{v_0}$ a corresponding eigenvector. Then the non-zero entries of $\mathbf{v_0 M}^n$ are proportional to λ^n, while the entries in the successive images of all other vectors grow at *at most* this rate. Since the 92 coordinate vectors (which we'll call **H**,**He**,$\cdots$,**U** in the obvious way) span the space, at least one of them must increase at rate λ.

On the other hand, our Chemical Theorem shows that the descendants of <u>each</u> of **He**,**Li**,$\cdots$,**U** increase as fast as <u>any</u> of them, and that this is at some rate >1, while **H** is a fixed vector (rate 1). These remarks establish our Theorem.

[We have essentially proved the **Frobenius-Perron Theorem**, that the dominant eigenvalue of a matrix with positive entries is positive and occurs just once, but I didn't want to frighten you with those long names.]

The Transuranic Elements.

For each number n≥4, we define two particular atoms:-

an isotope of **Plutonium** (*Pu*) : 31221132221222112112322211n
an isotope of **Neptunium** (*Np*) : 1311222113321132211221121332211n

For n=2 these would be Lithium (*Li*) and Helium (*He*); for n=3 they would be Tungsten (*W*) and Tantalum (*Ta*), while for n≥4 they are called the transuranic elements. We won't bother to specify the number n in our notation.

We can enlarge our 92-dimensional vector space by adding any number of new pairs of coordinate vectors **Pu**, **Np** corresponding to pairs of transuranic elements.

Our proof of the Ending Theorem shows that every digit 4 or more ultimately lands up as the last digit in one of the appropriate pair of transuranic elements, and (see the bottom left of Figure 1) that we have the decomposition

$$Pu \rightarrow Np \rightarrow Hf.Pa.H.Ca.Pu.$$

Now **Pu±Np** is an eigenvector of eigenvalue ±1 *modulo the subspace corresponding to the common elements*, since **Pu⟷Np** modulo that space. Because these eigenvalues are strictly less than λ in modulus, the relative abundances of the transuranic elements tend to 0.

So far, I can proudly say that this magnificent theory is essentially all my own work. However, the next theorem, the finest achievement so far in Audioactive Chemistry, is the result of the combined labours of three brilliant investigators.

The Cosmological Theorem.

Any string decays into a compound of common and transuranic elements after a bounded number of derivation steps. As a consequence, every string other than the two boring ones increases at the magic rate λ, and the relative abundances of the atoms in its descendants approach the values we have already described.

Proof of the Cosmological Theorem would fill the rest of ***Eureka!*** Richard Parker and I found a proof over a period of about a month of very intensive work (or, rather, play!). We first produced a very subtle and complicated argument which (almost) reduced the problem to tracking a few hundred cases, and then handled these on dozens of sheets of paper (now lost). Mike Guy found a simpler proof that used tracking and backtracking in roughly equal proportions. Guy's proof still filled lots of pages (almost all lost), but had the advantage that it found the longest-lived of the exotic elements, namely the isotopes of **Methuselum** (22333222l1n; see Figure 2). Can you find a proof in only a few pages? Please!

2233322211n (n>1)
223332211n
223322211n
222332211n
322322211n
1322133221ln
11132211232221ln
311322211213322lln
132113322112112322211n
*La.H.*12322211221121332211n
1112133221222112112322211n
*Sr.*3221132211221121332211n
1322211322212221121123222lln
111332211332113221122112133221ln
3123222.*Ca.*(*Li* **or** *W* **or** *Pu*)
1311121332
11133112112.*Zn*
*Zn.*321122112
131221222112
1113112211322112
311321222113222112
1321131211322113322112
11131221131112211322 2.*Na*
3113112221133122211332
Ho.Pa.H.Ca.Ac.H.Ca.Zn

Figure 2. The descendants of Methuselum.

The degree of λ.

Plainly λ is an algebraic number of degree at most 92. We first reduce this bound to 71 by exhibiting a 21-dimensional invariant subspace on which the eigenvalues of **M** are 0 or ±1. To do this, we define the vectors

$$\mathbf{v}_1=\mathbf{H},\ \mathbf{v}_2=\mathbf{He}-\mathbf{Ta},\ \mathbf{v}_3=\mathbf{Li}-\mathbf{W},\ \cdots\ ,\mathbf{v}_{20}=\mathbf{Ca}-\mathbf{Pa},$$

or, in atomic number notation,

$$\mathbf{v}_1=\mathbf{E}_1,\ \mathbf{v}_2=\mathbf{E}_2-\mathbf{E}_{73},\ \mathbf{v}_3=\mathbf{E}_3-\mathbf{E}_{74},\ \cdots\ ,\mathbf{v}_{20}=\mathbf{E}_{20}-\mathbf{E}_{91},$$

and also define

$$\mathbf{v}_{21}=\{\mathbf{Sc}+\mathbf{Sm}-\mathbf{H}-\mathbf{Ni}-\mathbf{Er}-3\mathbf{U}\}/2,$$

then observe that

$$\mathbf{v}_{21}\to\mathbf{v}_{20}\to\mathbf{v}_{19}\to\cdots\to\mathbf{v}_4\to\mathbf{v}_3\leftrightarrow\mathbf{v}_2,\ \mathbf{v}_1\to\mathbf{v}_1.$$

An alternative base for this space consists of the eigenvectors

$$\mathbf{v}_1 \text{ and } \mathbf{v}_3\pm\mathbf{v}_2$$

of **M** with the respective eigenvalues

$$1 \text{ and } \pm1,$$

together with the following Jordan block of size 18 for the eigenvalue 0

$$\mathbf{v}_{21}-\mathbf{v}_{19} \to \mathbf{v}_{20}-\mathbf{v}_{18} \to \cdots \to \mathbf{v}_5-\mathbf{v}_3 \to \mathbf{v}_4-\mathbf{v}_2 \to 0.$$

[This shows that **M** is one of those "infinitely rare" matrices that cannot be diagonalised. Don't expect to follow these remarks unless you've understood more of linear algebra than I fear most of your colleagues have!]

Richard Parker and I have recently proved that the residual 71^{st} degree equation for λ is irreducible, even when it is read modulo 5. We use the fact that the numbers in a finite field of order q all satisfy $x^q = x$ (since the non-zero ones form a group of order q-1, and so satisfy $x^{q-1} = 1$).

Working always modulo 5, we used a computer to evaluate the sequence of matrices

$$\mathbf{M}_0 = \mathbf{M},\ \mathbf{M}_1 = \mathbf{M}_0{}^5,\ \mathbf{M}_2 = \mathbf{M}_1{}^5,\ \mathbf{M}_3 = \mathbf{M}_2{}^5,\ \ldots,\ \mathbf{M}_{73} = \mathbf{M}_{72}{}^5,$$

and to verify that the nullity (modulo 5) of $\mathbf{M}_{n+2} - \mathbf{M}_2$ was 21 for $1 \leqslant n \leqslant 70$, but 92 for $n = 71$. Note that the 21 vectors of the above 'alternative base' are <u>eigenvectors</u> of $\mathbf{M}_2$ whose eigenvalues (modulo 5) lie in the field of order 5.

If the 71^{st} degree equation were reducible modulo 5, then $\mathbf{M}_2$ would have an eigenvector linearly independent of these with eigenvalue lying in some extension field of order $q = 5^n$ $(1 \leqslant n \leqslant 70)$. But then the eigenvalues ϕ of these 22 eigenvectors would all satisfy $\phi^q = \phi$, and the 22 eigenvectors would be null-vectors for

$$(\mathbf{M}_2)^q - \mathbf{M}_2 = \mathbf{M}_{n+2} - \mathbf{M}_2,$$

contradicting our computer calculations.

It is rather nice that we were able to do this without being able to write down the polynomial. However, Professor Oliver Atkin of Chicago has since kindly calculated the polynomial explicitly, and has also evaluated its largest root λ as

$$1.303577269034296391257099112152549 8$$

approximately. The polynomial is

$$\begin{aligned}
&x^{71} - x^{69} - 2x^{68} - x^{67} + 2x^{66} + 2x^{65} + x^{64} - x^{63} - x^{62} - x^{61}\\
&- x^{60} - x^{59} + 2x^{58} + 5x^{57} + 3x^{56} - 2x^{55} - 10x^{54} - 3x^{53} - 2x^{52} + 6x^{51}\\
&+ 6x^{50} + x^{49} + 9x^{48} - 3x^{47} - 7x^{46} - 8x^{45} - 8x^{44} + 10x^{43} + 6x^{42} + 8x^{41}\\
&- 5x^{40} - 12x^{39} + 7x^{38} - 7x^{37} + 7x^{36} + x^{35} - 3x^{34} + 10x^{33} + x^{32} - 6x^{31}\\
&- 2x^{30} - 10x^{29} - 3x^{28} + 2x^{27} + 9x^{26} - 3x^{25} + 14x^{24} - 8x^{23} - 7x^{21}\\
&+ 9x^{20} + 3x^{19} - 4x^{18} - 10x^{17} - 7x^{16} + 12x^{15} + 7x^{14} + 2x^{13} - 12x^{12} - 4x^{11}\\
&- 2x^{10} + 5x^{9} + x^{7} - 7x^{6} + 7x^{5} - 4x^{4} + 12x^{3} - 6x^{2} + 3x - 6
\end{aligned}$$

The International School for Advanced Studies

Trieste

D.W. Sciama

I am grateful to the Editor for inviting me to write about ISAS. This school was founded five years ago by the Italian Ministry of Public Instruction and is under the direction of Professor Paulo Budinich. It offers training for a PhD. degree in a number of theoretical sciences, namely astrophysics, particle physics, mathematical physics, solid state physics, biophysics and pure mathematics. I am in charge of the astrophysics sector, thereby illustrating the international character of the School. This character shows itself also in that the School offers grants to non-Italian students to cover their travel and living expenses (there are essentially no fees to pay) and in that the working language of the School is English.

Students normally spend their first year attending graduate courses on which they are examined, although those who have already taken Part III at Cambridge could expect to be admitted directly into the second year. At the end of this year the students write a Master's thesis of about 40 pages which is not required to be original. They then present a seminar based on the thesis, and are given an oral examination. The subject of the thesis is related to their research programme, and preparing for it is not intended to be a significant distraction from their PhD. Nevertheless, because of the first year concentration on courses, most students take four years to complete their PhD. and they can expect their ISAS grant to be extended for a fourth year.

Students who would be eligible for an SERC grant to study for a PhD. in the U.K. may also apply to SERC for a similar grant to study at ISAS (and two students in the astrophysics sector currently have such grants). In this case the student could expect to be supported for a fourth year by ISAS.

I cannot write with authority about the interests of all the sectors, but in astrophysics we have a particular (but not exclusive) interest in cosmology, relativistic and high energy astrophysics and galactic dynamics. We expect to add several professors and associate professors to the astrophysics staff in the next two years, and this should lead to a broadening of our interests.

One of the attractions of ISAS is that it is next door to the International Centre for Theoretical Physics. This twenty-year old Centre is directed by Professor Abdus Salam. It is a meeting place for physicists from third world countries and those from developed countries, and houses many international workshops and conferences in all branches of physics. ISAS shares many of its facilities, and our new building is just one minute's walk from the main building of the centre. I have been appointed a consultant to the centre, with responsibility for their work in cosmology and astrophysics, and to liaise with the Observatory of Trieste. ISAS also has close links with the Observatory and some of our PhD. students spend a considerable fraction of their time on observational work.

The joint campus is located five miles north of Trieste close to the sea and with beautiful views of the bay and the hills lying immediately behind. In addition to the buildings on the campus, the Centre is renting a first-class hotel which overlooks the sea at Grignano ten minutes walk away and next door to the romantic castle at Miramare. This hotel provides bedrooms for short term visitors, a lecture room and an excellent restaurant, where one can dine on an open-air terrace in the summer. There is also a somewhat cheaper cafeteria in the main building of the Centre. All these amenities are available to ISAS students.

Trieste is a city of some 300,000 people and has had an eventful history resulting from its strategic position at the northern end of the Adriatic. It reached a peak of prosperity in the eighteenth and nineteenth centuries when its port was developed to serve the needs of the Austro-Hungarian Empire and many of its finest buildings date from these times. To the south is Istria where there are still many traces from centuries of Venetian rule; to the north and east is a high limestone plateau called the Carso which has characteristic vegetation and many caves and underground rivers. Much of the natural hinterland now lies within Yugoslavia but there are normally few border restrictions and so visiting is easy. Fifty miles to the north are the Julian Alps where there are extensive opportunities for mountaineering and skiing, while the coastline has many bathing places. Venice is two hours away by train and, for longer excursions, the whole of Italy and central Europe can be reached quite easily.

The combination of scientific facilities and location make ISAS an excellent place to do a PhD. If you are interested please let me know. The address is Strada Costiera 11, 34014 Trieste, Italy.

Problems Drive

Paul Smith
introduction by Bob Dowling

Each year the Archimedeans hold a little get-together when every convention on human rights is thrown to the wind. The Problems Drive this year was a combination of sadism on the part of last year's winners (who set this year's puzzles) and masochism on the part of the 22 foolhardy souls competing, six of whom had made the trek from Oxford to be there. There they were subjected to some of the hardest puzzles the Drive had ever witnessed. Surviving not only the mental challenge of the problems but also some of the foulest catering the Archimedeans had ever created these worthies battled on to the end for the presentation of the highly prestigious awards:

To the winners, a bottle of port.

To the creators of the most original or unusual answers, a mango.

To the team with the lowest score, the Wooden Spoon.

The questions will not be so difficult this year - we promise! The catering will be palatable - honest ! Pigs may fly - really !

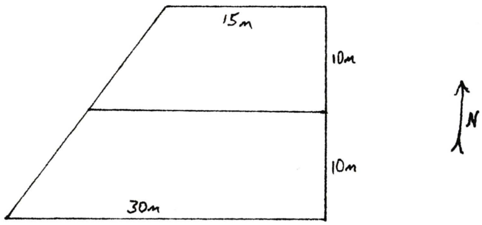
1) Come In Number 61...

Boats numbered 1 to 30 inclusive are sailing on a pond. A wicked witch has put a spell on the boats so that when she calls out any number x from 1 to 60 (inc.) all the boats with number dividing x will sink. However, each time a number x is called out, at least one boat with number strictly less than x must sink.

What is the minimum value of $\sum x_i$ for a sequence $x_1, x_2, \ldots, x_r$ such that all the boats will be sunk?

2) Fencing Lessons

Here's a map of the gardens of Mr.Smith and Mr.Jones. How far south must the dividing fence be moved (it will have to be lengthened) in order to give the two gardens the same area ?



3) Alphabet Soup

Find the next two terms in each of the sequences, all derived from the series ONE, TWO, THREE, FOUR...

i) 5 , 15 , 18 , 21 , 22 , 24 ...
ii) 34 , 58 , 56 , 60 , 42 , 52 ...
iii) 10 , 8 , 15 , 15 , 17 , 15 ...
iv) 42 , 4 , 22 , 45 , 32 , 3 ...
v) 10 , 205 , 5 , 318 , 19 , 557 ...

4) Colour By Numbers

In a game Albert draws (path-connected, finite) countries on a (plane, infinite) surface. After each country is drawn, Barry must colour it in, subject to the restriction that no two bordering countries may have the same colour. Albert wants to force Barry to use 6 distinct colours. How many countries are needed for this if each new country must border at least one existing country ?

5) Where Have All The Flowers Gone ?

I, J, R, S are two boys and two girls. On one St. Valentine's Day each boy gives each girl a flower and vice-versa. (Isn't that sweet ?) So exactly 8 flowers a, b, c, d, e, f, g, h are involved. The following is known :-

i) g and h were swapped.
ii) I neither gave g nor received a.
iii) The person who gave f is the same sex as S.
iv) h was received by a boy.
v) The person who received e gave a flower to I.
vi) R neither gave nor received g.
vii) a and b were given by different people.
viii) Neither girl received b.
ix) d and e were given by people of the same sex.
x) a and b were received by different people.
xi) I gave a flower to the person who received g.
xii) d and f were either given by the same person or received by the same person.
xiii) A certain person of the same sex as R was involved with both d and h.

Who are the girls ?

Who gave which flowers to whom ?

6) A Fly In The Ointment

Freddy the fly crawls along the inside of the edges of a cube and wishes to go along each edge exactly twice, once in each direction. (No I don't know why. Ask Freddy.)

Give a sequence of instructions, telling Freddy whether to go right (R) or left (L) at each vertex he reaches, so that he achieves his goal.

7) No-go Situation

A game is played on a 19x19 grid. The grid is initially empty. One player has a collection of identical black pieces; the other has white pieces. As the first move, player I puts a black piece on any grid point. Then player II puts a white piece on any vacant point. Then player I puts another black piece on any vacant point.

How many essentially different positions (so that no symmetry of the square sends one to another) are possible at this stage ?

8) Gnomadic Gnomes

Mrs. P. has 4 gnomes, all of different colours. Each gnome is sometimes found in the back garden and sometimes in the front garden. Each night exactly one gnome moves from one garden to the other. When a gnome moves it is so tired that it cannot move again on either of the following two nights.

What is the maximum number n of consecutive days such that Mrs. P. could find n different collections of gnomes on those n days ?

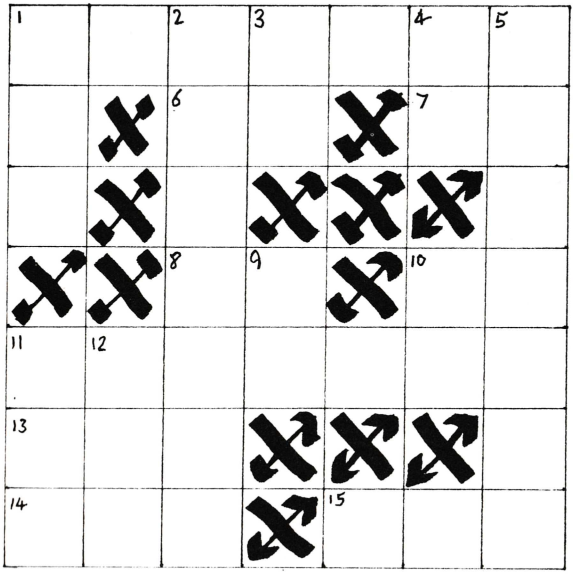
9) Trancend-dental Meditation

In a bizarre religious ceremony there are infinitely many priests, one assigned to each non-negative integer. At 1 hour intervals they all go into a large (very large!!) room. On each visit priest #r chants "OM MANE PADME HUM" if and only if on any of the preceding n visits to the room the number of priests chanting was exactly r. n is a positive integer chosen in advance by the High Priest. The first visit, when no-one says anything, is on Friday at 1pm. Priest #3 wants the last time that he says "OM MANE PADME HUM" to be no later than Saturday at 4pm so that he can slip off unnoticed to a dentist's appointment. What is the least value of n that will make him late ?

10) View From A Paddy Field

Paddy stands at a point P and looks out over a circular field which has a fence across it. In whichever direction Paddy looks out over the field, he sees a nearest point of the field Q, a point R of the fence and a furthest point of the field S in that direction. Also the ratio PQ.RS/PS.QR is 1 in all such directions. Describe the shape and the position of the fence in relation to Paddy and the field.

11) Cross Number



Across:

1 $(14a - 10)^3$
6 a prime
7 divides 1d
8 a prime
10 14a/7
11 a cube
13 twice a prime
14 middle digit is 0
15 a prime ; 1d reversed

Down:

1 divisible by 23
2 $(14a + 4)^3$
3 divisible by 11
4 a prime
5 = 1a
9 divisible by 5
10 divisible by 4
11 divisible by 7
12 divisible by 3

12) Just Not Cricket

Find the next two terms of :-

i) 1, 7, 11, 17, 19, 41, 47, 49, 61, 67, 71 ...
ii) 9, 18, 27, 31, 22, 31, 40, 49 ...
iii) 1, 3, 13, 69, 431, 3103 ...
iv) 529, 667, 713, 851, 943, 989 ...
v) 72.35, 63.62, 59.43, 73.65, 67.78, 68.04, 51.37 ...

(In (v) the points are decimal points not multiplications.)

Token Cynic

(Vice-Presidential mutterings.)

Has anyone considered how much CPU time has been spent throughout the world plotting pretty pictures of the Mandelbrot set ? One undergraduate I have heard of has spent hours single-handedly. The overall picture does not bear thinking about - unless of course you are puzzled about why the usage of the computer is higher at midnight than it is at noon .

Overheard at a supervision :

Supervisor : Do you think you understand the basic ideas of Quantum Mechanics ?

Supervisee : Ah! Well,what do we mean by "to understand" in the context of Quantum Mechanics ?

Supervisor : You mean "No" , don't you ?

Supervisee : Yes.

The Tautology prize goes to a lecturer who uttered the gem :
" If we complicate things they get less simple."

This year's Modesty award goes for a phrase spoken by a lecturer after a rather difficult topic had just been introduced.
" You may feel that this is a little unclear but in fact I am lecturing it extremely well."

Overheard at last year's Archimedeans' Garden Party :
"Quantum mechanics is a lovely introduction to Hilbert spaces !"

Spoken by a category theorist :
" Category theory doesn't make everything trivial but when something is trivial it shows that it's trivially trivial ."
(I am told this is a garbled version of Peter Freyd's classic remark "Category Theory shows that that which is trivial in Mathematics is trivially trivial." but it sounds good anyway. - Eddy)

A Senior mathematician was asked which language he used for some of his computing . He replied that he used a very high level language:

RESEARCH-STUDENT

A Storm in a Teacup

Paul Matthews

If you wander into the DAMTP coffee room at about eleven o'clock, you will see small groups of mathematicians staring into cups containing a dull brown liquid. They are, of course, studying fluid mechanics, and the so-called 'coffee room' is simply a thinly disguised extension of the fluid mechanics laboratory in the basement. The questions on their minds as they gaze thoughtfully into the cup and rotate the liquid with a spoon are:

What is the form of the flow in the cup?

Is it purely a rotation, or are there more complicated motions?

What is the shape of the surface ?

How does the milk or sugar become mixed ?

Is the flow turbulent ?

Let us look at some of these questions of great practical importance. Consider first a rotating flow in an infinitely long cylinder of radius a, using cylindrical polar coordinates. Suppose that the flow is in the θ-direction only, and independent of θ and z (fig 1.). Then the equation of motion for the speed u is

$$\partial u/\partial t = \nu(\nabla^2 u - u/r^2)$$

where ν is the kinematic viscosity. (This equation can be looked up in a standard textbook e.g. Batchelor, An Introduction to Fluid Dynamics, p.602.) Notice that this is simply the diffusion equation for the vorticity, $\boldsymbol{\omega} = \nabla \times \mathbf{u}$, so vorticity diffuses through the fluid with diffusivity ν.

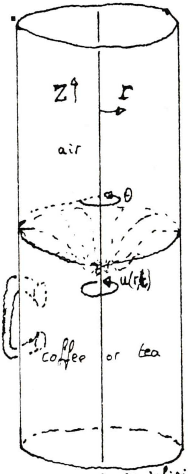


fig 1 an infinitely tall cup

The solution can be separated in r and t ; set $u = u_0 f(r) g(t)$,

then $\dot{g}/g = \nu(\nabla^2 f/f - 1/r^2) = -c$

Since the l.h.s. is a function of t only, and the r.h.s. is a function of r only, c is a constant, to be determined later by the boundary conditions. Then $g = g_o\exp(-ct)$, and f obeys

$$f'' + f'/r - f/r^2 + cf/\nu = 0$$

Here we immediately recognise our dear old friend Bessel's equation:

$$r^2f'' + rf' + (\frac{c}{\nu}r^2 - 1)f = 0$$

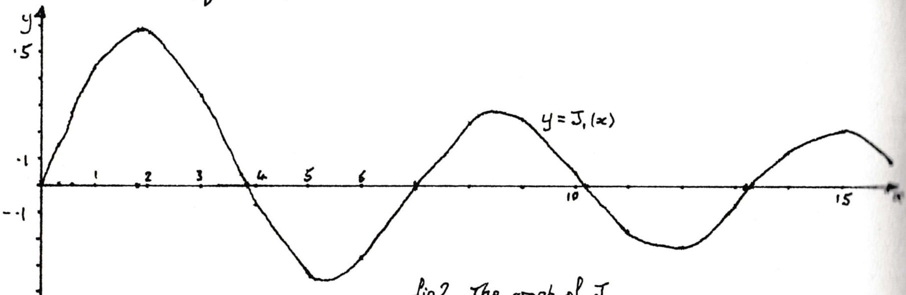


fig 2 The graph of J_1

The solution which is finite at r=0 is $f(r) = J_1(r\sqrt{\frac{c}{\nu}})$, where J_1 is the Bessel function of order 1, shown in fig. 2 for those who (like the editor) may not be intimately acquainted with it. We can now determine the decay rate c from the condition that u must vanish at r=a. The first three zeroes of J_1 occur at 3.83, 7.02 and 10.17 . If we call the zeroes α_i, then

$$f_i(r) = J_1(\alpha_i r/a) \ , \ i=1,2,...$$

form a set of solutions. Since Bessel's equation is self-adjoint $\left[\text{it can be written } \frac{d}{dr}(rf') + (\frac{c}{\nu}r - \frac{1}{r})f = 0\right]$ these solutions are orthogonal, and any well-behaved initial conditions can be thought of as a Bessel series, just as the motion of a plucked string can be thought of as a Fourier series. The constant c is then determined by $\sqrt{\frac{c}{\nu}} = \alpha_i/a$, i.e. $c = \nu\alpha_i^2/a^2$, and the full solution for axisymmetric flow in an infinitely long cylinder is

$$u = \sum_1^\infty a_i J_1(\alpha_i r/a)\exp(-\nu\alpha_i^2 t/a^2)$$

where the coefficients a_i are determined by the initial conditions. Thus the higher modes decay faster than the lower ones, and at large times the i=1 component dominates.

We can now calculate the shape of the free surface for this first mode. This is determined by the condition that the net force, gravitational plus centrifugal, on a particle on the surface must be normal to the surface. If the surface is z(r), then

$$\frac{dz}{dr} = u^2/gr = \frac{u_o^2}{gr}J_1^2(\alpha_1 r/a)$$

which can be written

$$\frac{dz}{dr} = \frac{u_o^2}{g}\left\{-J_1(r')J_1'(r') - J_0(r')J_0'(r')\right\}$$

where $r' = \alpha_1 r/a$, since the Bessel functions obey

$J_0'(x) = -J_1(x)$ and

$J_1'(x) = J_1(x) - J_1(x)/x$.

Thus we can integrate and obtain

$$z(r) = \frac{-au_o^2}{2g\alpha_1}(J_1^2(\alpha_1 r/a) + J_0^2(\alpha_1 r/a)) + \text{constant}$$

for the shape of the surface. For comparison, the shape would be parabolic if the fluid were in solid body rotation.

Let us now evaluate the decay rate c for a cylinder of radius 3cm, considering the lowest mode. The viscosity ν is highly temperature dependant, being .018 cm^2s^{-1}at 0^oC, .010 at 20^oC and .003 at 100^oC. Thus freezing water is six times as sticky as boiling water. [How to tell a hot cup of tea from a cold one: stir them both and the cold one will stop sooner.] If we take ν=.005 cm^2s^{-1}, corresponding to a temperature of 56^oC, then

$$c = \nu\alpha_1^2/a^2 = .005\text{x}3.83^2/3^2 = 8.15\text{x}10^{-3}s^{-1},$$

or, if we write $c=1/t_o$, where t_o is a characteristic slowing down time, t_o = 123s. Two minutes seems rather large for the time taken for the speed to drop by a factor e^{-1}=0.37. Indeed, experiments conducted in DAMTP suggest that t_o is around 20s for the teacup.

Clearly, there is something wrong here - the infinite cylinder is not a good model of a teacup. We could have realised this from the diffusion equation, since the only timescale in the problem is a^2/ν = 1800s which is much too large. One might try to incorporate the influence of the bottom of the cup by allowing u to vary with z, with u=0 at z=0. This makes very little difference to the answer and is still wrong. The influence of the bottom is more complicated than this, and, to see why, we must look at the physics of the situation or the full equations describing the motion.

Physically, the rotating fluid sets up a radial pressure gradient given by

$$\frac{dP}{dr} = \rho u^2/r$$

which provides the centripetal force to support the circular motion. Since the vertical pressure gradient is simply $-\rho g$ in the absence of any vertical motion, this same radial pressure gradient exists near the bottom of the cup, where the circular motion is much less, due to the viscous drag. Thus there is a net inward force on the fluid near the bottom and a secondary motion is set up, as shown in fig. 3. This explains why the tea leaves, biscuit crumbs, etc. always seem to end up in the middle of the bottom of the cup. This secondary motion makes the fluid slow down more rapidly, because the bottom layer which loses its momentum is constantly being exchanged - the process is a combination of diffusion and convection rather than pure diffusion .

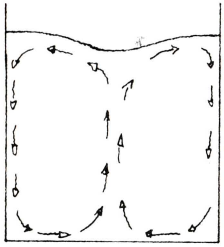


fig 3. the secondary motion in cross-section.

The full equations for the flow, assuming only that the flow is axisymmetric, are

$$\frac{\partial v}{\partial t} + \mathbf{u}.\nabla v - \frac{u^2}{r} = \frac{-1}{\rho}\frac{\partial P}{\partial r} + \nu\left(\nabla^2 v - \frac{v}{r^2}\right)$$

$$\frac{\partial u}{\partial t} + \mathbf{u}.\nabla u + \frac{vu}{r} = \nu\left(\nabla^2 u - \frac{u}{r^2}\right)$$

$$\frac{\partial w}{\partial t} + \mathbf{u}.\nabla w = \frac{-1}{\rho}\frac{\partial P}{\partial z} + \nu\nabla^2 w - g$$

$$\frac{\partial w}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(rv) = 0 \qquad \text{conservation of mass}$$

where $\mathbf{u} = (v,u,w)$ in (r,θ,z) coordinates,

$$\nabla^2 = \frac{\partial^2}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right),$$

P is the pressure and g is the acceleration due to gravity.

A prize of a cylindrical mug for conducting further experiments in this field is offered to anyone who can produce a full solution for arbitrary initial conditions to these coupled nonlinear partial differential equations on a postcard. (Not taking any risks with this one ! - Eddy)

One way in which the problem can be tackled is by the use of boundary layer analysis. Suppose that $\mathbf{u}$ is given approximately by our previous J_1 solution except in a thin layer of thickness δ near the bottom in which viscosity is important in slowing down the fluid. From the u equation, this occurs when

$$\mathbf{u}.\nabla u = O(\nu\nabla^2 u)$$

The l.h.s. is of order U_o^2/a where U_o is the magnitude of u outside the boundary layer, and the r.h.s. is of order $\nu\frac{U_o}{\delta^2}$, so $\delta = \left(\frac{\nu a}{U_o}\right)^{1/2}$

which is about 1mm for U_o=3cm s^{-1}. Note that δ is small compared with a or the height of the cup, so our original supposition is consistent and the approximation of ∇^2 by $1/\delta^2$ is reasonable. Now consider the v equation. The dominant terms, omitting detailed justification, are the driving pressure gradient of order U_o^2/a and the viscous resistance $\frac{\partial^2 v}{\partial z^2}$. Equating these two terms, with v=0 at z=0 and z=δ gives

$$v = \frac{U_o^2}{2a\nu} z(z-\delta) = O\left[\frac{U_o^2}{a\nu}\delta^2\right] = O(U_o)$$

This breaks down near r=0 and r=a, where the horizontal gradients and the vertical velocity w become important. Note that v is of the same order of magnitude as U_o in the boundary layer, though the above argument suggests that it would be at most $U_o/8$. The volume flux through the boundary layer caused by the secondary motion is $O(va\delta) = O(\sqrt{(U_o \nu a^3)})$. If the height of the cup is h, then the timescale for all of the fluid to pass through the boundary layer and so lose a significant amount of its momentum is equal to the total volume of the cup divided by the volume flux through the boundary layer. Thus the slowing down time is

$$t_o = a^2 h/\sqrt{(U_o \nu a^3)} = h\left[\frac{a}{\nu U_o}\right]^{1/2}$$

This is the geometric mean of the diffusion timescale based on the cup height, h^2/ν , and the rotation timescale a/U_o . For h=5cm, U_o=3cm s^{-1}, t_o is 71s, which is better than our first answer though still rather too large. Notice that t_o is proportional to $U_o^{-1/2}$, so if this were the dominant mechanism for slowing down the flow, one would expect

$$\frac{dU_o}{dt} \propto \frac{U_o}{t_o} \propto U_o^{3/2},$$

giving $U_o = 1/(A+Bt)^2$ for the time-dependence of U_o , rather than the exponential time dependence which we found for the infinite cylinder.

The Society

The Last Recorder

For many years now the secretary's report in *Eureka* has claimed that the society has had a good year. It has, in point of fact, had only mediocre years as long as I can remember. That said, this year has had a few things to be said for itself. Whether we are on an upturn, or just a squigle in the variation from year to year, will depend on what people do next year; but for that look elsewhere in this issue.

This year, there is no secretary's report because the retired Recorder has a better idea of what has happened than the secretary. The year has seen its usual smattering of talks by the Great, the Famous and some others. Perhaps the most notable of these was a talk by Dr. Edward de Bono, with a very long title, in which some of us were awakened to some of the faults in the way we think.

We also held our usual collection of informal events, ranging from a Graduate Research Opportunities Evening to a highly successful garden party after exams. The committee decided not to organise a ramble through Cambridgeshire's countryside, so the old guard, masquerading as a silly sandwiches subgroup of the New Pythagoreans, organised one instead. I think it got rained off. This summer was like that.

One of the highest points of the year was the Triennial Dinner, at which we celebrated the Society's fiftieth anniversary. This proved highly enjoyable. Professor Sir Hermann Bondi, who had recieved a rather curious present from the President, gave a delightful speach ending in a toast to *the Society*, Professor Conway, who had brought some impossible drinking vessels with him, employed Archimedes' dying words "*Get out of my way*" as a toast to Mathematics, and the President did his best not to fluff his first after dinner speach, which ended with a toast to *our elders and betters*, since we always have one or the other.

We have only just begun to make some attempts to broaden the scope of Archimedean activity, with our two discussion meetings at the end of the Michaelmas term, one more successful than the other.

It is my humble opinion that the society is poised at a critical point in its history; either we revive now and become a worthwhile society, or we sink back into mathematics. If people continue to believe that the Society is only about maths, so that only the people interested in maths take an active pert in the Society, then belief will again become truth. Think on it. I look forward to seeing who turns up to events in the new year; I hope some of you are interested in more than just what you came here to study.

rmj

16.XII.85

Situations Vacant

Eddy

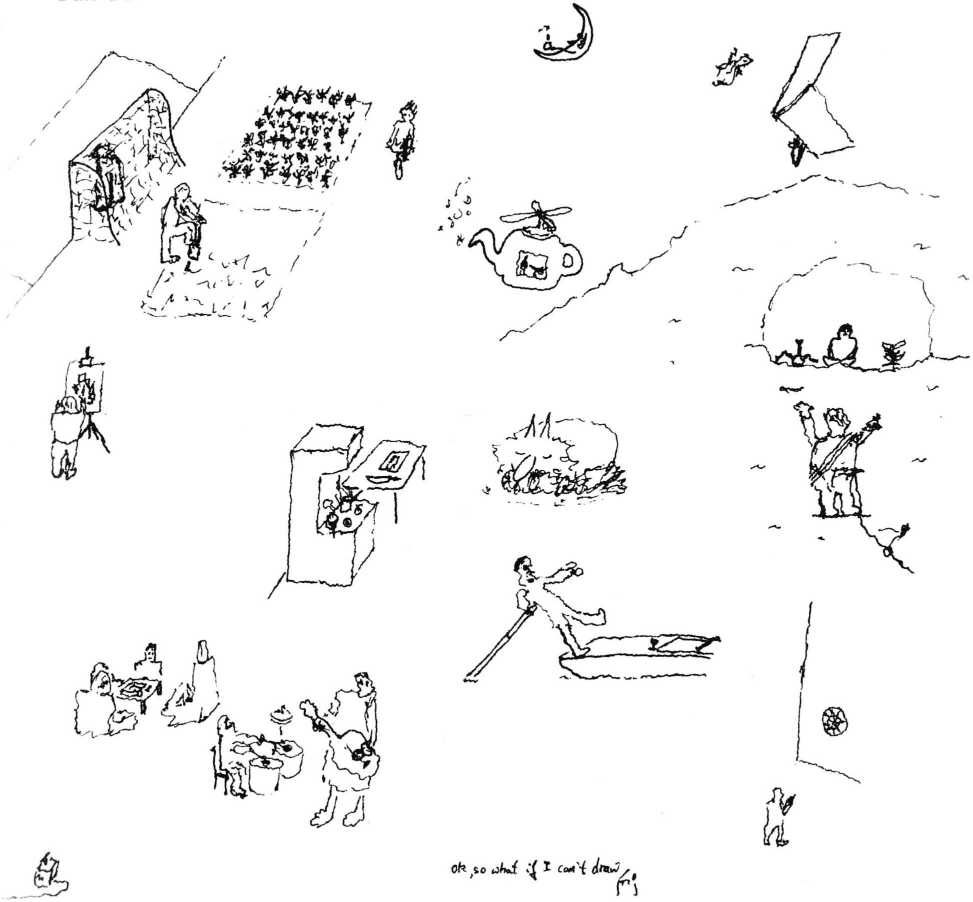
The editor of *Eureka* should be a first or second year, not a Part III student or research student. Third years are just about excusable, if they can bear the damage it may do to their finals result. Thus I am seeking to recruit someone as my successor, preferably a present first year. The job involves a fortnight or so, ideally in September but more often over Christmas, of typing and preparing the articles for publication plus sporadic hours throughout the year of conning people into writing articles for you. This is the important bit, not the typing for which you should be able to get some assistance. The editor needs to be able to get people to write articles which will be of interest to the readership; the fact that readers are mathematics students does not mean that they are interested only in mathematics, indeed even that they are particularly interested in mathematics as far as I can tell. In recent years there has been a tendency towards heavy mathematics by the great and the mighty, leading one potential editor to turn the job down on the grounds that it was more learned than she could handle. My hope is to change precisely that; it is the journal of an undergraduate society, so undergraduates should be interested in it and able to understand it. And if the mathematics that undergraduates can understand doesn't make interesting reading, try something else; a discussion of the merits of unilateral disarmament, or perhaps of the philosophical consequences of some aspect of modern science. If you believe that the subject is going to interest enough readers, then it will do; and given that *Eureka* has been known to carry articles of interest to only about a dozen mathematicians, I see no reason why my successor should be unwilling to stray off the path of mathematics in pursuit of what interests *Eureka*'s readers. The same sort of argument applies to anyone wishing to write articles for *Eureka*, and even to anyone standing for office in the Society. A secretary who does not recognise the diversity of interests among the Society's members will do a lousy job. An Entertainments Manager who made the same mistake would be a disaster.

Come March, there will be elections for the officers of the society for the coming year; the posts to be filled are listed inside the front cover along with the name of the current officers. There is no reason, aside from bad precedent, why a first year should not fill any of these posts, and in that I *do* include the Presidency. The society has several non-elective posts as well, such as the Bookshop Manager, the Editor of *Eureka*, the business managers of *Eureka* and QARCH, and many more; some of these will be vacant, notably *Eureka* Editor, and anyone interested should talk to the current holder of the post about what it involves, and optionally to other officers for another angle on the job; and other people intending to stand for office so as to get some idea of who you might be working with.

If noone interesting comes forward, the Society will slither back into oblivion. I hope you like that idea as little as I do.

In the mean time, I have two suggestions that I have nurtured for some time, which I think someone might enjoy picking up; I will attempt to ensure that the Society helps anyone who choses to do so. The first is football (or any of a dozen other sports); I am aware of there being a lot of mathematics students who indulge in soccer, and in the days of my second year, when I still had time to play, I ran into an engineer who suggested a match between the engineers and the mathematicians. It occurs to me that this might be highly enjoyable if someone could muster the time to organise it. Any volunteers please see me, Eddy, if only to recruit a slightly out of practice sweeper and perhaps get some publicity. The second is food. We do occasionally have lunch as part of our Saturday meetings, but I was thinking more in terms of a weekly lunch organised in such a way that it was attended by more than just the people who come to the speaker meetings and discussions, so that another section of the membership could get something out of the society, namely a social meeting with a cheap lunch. Anyone willing to organise this will be most welcome. Equally,...

Anyone with some favourite idea for something they would like to do, if they can persuade me that there are more than about a dozen members interested in it, I will help them to the best of my ability to get it going, provided they are willing to run it.



Was Descartes' mathematical physics Mathematical Physics?

Piers Bursill-Hall

Rene Descartes is one of those "heroes" of the history of science that everyone - simply everyone - has heard about. He is remembered in elementary mathematics texts as the discoverer (or inventor, depending whether you think maths is discovered or invented) of analytic geometry, and so of [Cartesian] coordinates, and in the sciences in general as one of the central figures in the great reform and change of scientific foundations, concepts, methods and goals that are all lumped together under the general title of the "Scientific Revolution". Cartesian coordinates, analytic geometry, 'cogito ergo sum', the "mechanical philosophy" and that sort of thing are all part of the commonly held folklore of history of maths and science. And yet, for all that he is thought to be one of the central figures in the development of early modern science, and a crucial influence on later 17th century figures like Huygens, Pascal, Leibniz and Newton, he appears in some ways to have accomplished remarkably little beyond this algebraic geometry (the consequences of which were generally unappreciated for most of the 17th century... which he was not the first or the only one to discover, anyway). Was Descartes really a clod?

Descartes was born in France in 1596, and educated at the very avant-garde Jesuit college at La Fleche - where up-to-date physics and mathematics were taught along with the more traditional curriculum. Financially independent, he joined as a 'gentleman volunteer' the army of Prince Maurice of Nassau, and in his leisure time over the late 1610s and early 1620s began to map out "an entirely new science" (analytical geometry), along with a whole new vision of what science - and knowlege of the physical world - was about. Nothing if not amazingly immodest, Descartes was of the opinion that he - and he alone - could completely reform all science and that his new methodology alone would reveal all true knowledge.

Descartes, ever so slightly optimistically, thought that his new scientific method and his ideas as to how the world worked would unravel in a fairly short time all the mysteries of the universe, answering all scientific questions with complete certainty. The key to this remarkable new scientific method was Descartes' extension of mathematical methods of study to all natural phenomena. He claimed this would be able to make all science like mathematics in the necessary truth and certainty of its conclusions, and so his method, his mathematico-physics or mathematical physics would give a complete and rational understanding of all physical phenomena. It will come as no surprise to you to learn that this programme was not successful: perhaps Descartes ***was*** a clod.

The promise of Descartes' science may not have been quite fulfilled - indeed, as he proposed it, it was simply unworkable anywhere - and analytic geometry didn't turn out to be the unifying universal mathematics he claimed it was... but all the same, it is really quite difficult to overestimate the importance of the ideas or changes associated with Descartes. Algebra - or something vaguely like algebra, such as generalised arithmetical-equation solving - had been around for a long time; it had been studied more or less since the early Babylonian and Egyptian mathematicians of - say - 1000-2000 B.C., and was much developed by the Arabs in the 9th - 13th centuries A.D., whilst geometry, of course, was the glory of ancient Greek mathematics from the 5th century B.C. to the 5th century A.D. Both algebra and geometry had been much studied by Arab, medieval and Renaissance mathematicians, and in the century before Descartes considerable (no, spectacular) progress had been made in both subjects. Bringing the two together, seeing that algebra and geometry were, in some sense, about the same sorts of "things" was an amazingly fruitful thing to do. The "algebraicisation" of mathematics (by which, by the way, is meant the slow but steady seepage of algebraic techniques & methods, concepts, language and intuitions into the rest of mathematics; a long & complicated process over the late 17th, 18th and early 19th centuries) is perhaps the single most important underlying force that spurred on the development of early modern mathematics from its classical, geometric predecessors, and it was learning to think and talk about mathematical entities in an algebraic manner that led, in very large part, to the state where in the early or mid 19th century a series of mathematical discoveries brought about the revolution in foundations and ideas that lies at the beginnings of modern mathematics. So putting algebra and geometry together was the beginning of a pretty important development; 10 out of 10, René.

Indeed, it is not widely appreciated, but Descartes appears to be the inventor of one of the most profound and powerful methods of proof, ranking with classical methods of proof like *reductio ad nauseam*, Proof by Misdirection, or Proof by Non-Existent Reference. For those who might not be familiar with these, proof by *reductio ad nauseam* works thus: the lecturer denies the result that is to be proved, and starts off writing down in a random order all the consequences of the counter-proposition that s/he can think of (preferrably in a quiet, dull monotone) in crabbed handwriting all over the blackboard. When all the students are asleep (or at least bored and distracted, and not taking notes - this is a strong condition), the lecturer cleans the blackboard, announcing: "Thus, with this contradiction, we have established the result as desired." At this point the audience will wake up, and decide to get the last part of the proof from someone else (the assumption of student inertia weakens the method, it is better to have the **whole** audience asleep). Proof by Misdirection proceeds by writing a claim of the form A -> B in as convoluted a fashion as possible. Then B -> A is proved, which is usually easier and will satisfy most audiences. If there is ample time to fill in, the countably infinite analogue may be used - Proof by Convergent Irrelevancies. Proof by Non-Existent Reference is simple and elegant: the proof is claimed to be in Korner, 3rd ed, p.597. Either there is no 3rd edition of this (now out of print) text, or p.597 is in the index.

Descartes was a great pioneer in these methods of 'extended' proof theory. At the end of his *Geometrie* (which is really about the relationship between algebra and geometry), he admits that he has been a little sketchy and obscure, but, he says, this was done intentionally so as "not to deprive the reader of the joy of discovery": perhaps one of the earliest examples of 'proof left as an exercise for the reader', now formally stated as "we leave as a trivial exercise for the reader the proof that..."

In the physical sciences Descartes' new ideas seem, if anything, even more significant: it was he who (it is sometimes said) introduced the idea that nature works like a machine -

> "I have described ... the whole visible world as if it were only a machine in which there was nothing to consider but the shapes and movements of its parts..."

with regular, rationally understandable (and mathematical) laws governing phenomena, rather that by mysterious, sometimes occult, and unexplained qualities or *forces* (we shall quietly ignore that Newtonian physics, as he put them forward in the *Principia* (1687), ran the universe on the unexplained *force* of gravity acting across space...). For Descartes, things don't "do" things because it is their proper place to do them, or because it is what they are supposed to do, or because of their special *nature*; macroscopic phenomena that we can sense occur only because of the physical collisions of microscopic particles or corpuscles. The world, Descartes claimed, was made up of inert matter whose only property was that it could move, and completely non material things like thoughts and ideas that could have no physical manifestation. The material world consisted of a plenum of various kinds of microscopic corpuscles with even smaller ones filling in the interstices of the larger, leaving no space unoccupied. This idea was to turn out to be quite extraordinarily fruitful, too, and even if Descartes' corpuscles aren't quite our modern atoms, they seem to be a considerable way along the path, and clearly a Good Thing.

But perhaps most important of all, Descartes had the idea that you can mathematise physics - that phenomena can be meaningfully and correctly represented by mathematical models, and that thinking about the world mathematically is the best and the most certain way to know about it because a mathematically demonstrated physics would be completely incontrovertible. It sounds good, and if no one had thought of it before, it surely must stand as one of the greatest innovations in science. So, it is supposed, not only did Descartes invent the "mechanical philosophy" or the "mechanical world view" (that later gave rise to Newtonian mechanics), but he invented the very idea of mathematical physics... and with this, his thinking formed the centre of the new ideas about science that are held to be the origins of modern science.

That's the way the story goes, and its fine in as far as it goes, with one minor problem: more or less none of it is quite true. Which is difficult.

Firstly, for all that Descartes used algebraic techniques in geometry in some new ways and saw the relationship between equations & curves and solving equations & constructing curves in a deep way, he was not the only one to discover analytic geometry nor even the first to see algebra and geometry as deeply related. *In extremis* it could even be argued that some of the early theorems of Euclid's *Elements* Book V show some sort of algebra-geometry link because they are bizarre, strange, and difficult geometric constructions to understand... until one sees them as the constructions of solutions to equations. Consider, for example, Book V,6:

> If a straight line be bisected and a straight line be added to it in a straight line, the rectangle contained by the whole (with the added straight line) and the added straight line together with the square on the half is equal to the square on the straight line made up of the half and the added straight line.

It may be difficult to make much sense of such a geometric construction until you interpret it as:

$$ax + x^2 = b^2$$

After which, you can see that this has been shown to be true. Similarly for the other examples like this: this is sometimes thought to be a "geometrical algebra" (whatever that might mean) because these theorems somehow seem to be about algebraic things. But this is a little dubious, since such algebraic notions as we see in V,6 were certainly **not** in Euclid's mind; it is just *us* using our algebraic intuitions to see some geometry in the guise of (what we now know to be) its underlying algebraic meaning. But a little later - well, 1200 years later actually - the 9th century Arab mathematician al-Khowarizmi wrote one of the earliest treatises on the art of equation solving (the first words of the title were *al-jebr*, and that is where the word algebra comes from); in this work al-Khowarizmi gives the rules for solving equations such as the various cases of the quadratic, using a calculating recipe equivalent to the quadratic formula. In the second section of the book he proves these recipes - using geometric constructions! This odd conflation of an algebraic calculating method proved by a geometric analogue or interpretation of the method is only the tip of the iceberg of the Arabs' mixing of algebra and geometry; in Omar Kyayyam's *Algebra* he did more of the same, but to the extent of writing "no attention should be paid to the fact that algebra and geometry are different in appearance. Algebras are geometric facts which are proved."

But of greater significance, it should be noticed that "algebraicising" geometry (in some sense) was very much in the air at the time. Two slightly older contemporaries of Descartes - the Englishman Hariot and the Frenchman Vieta (working about 1580-1600) had gone a long way towards analytical geometry, and Descartes' contemporary Fermat (working 1620-50s) is certainly as much the discoverer of it as Descartes. Vieta (and Hariot) saw that algebra reasons about quantities more general than numbers, about generalised "quantity" in much the same way the 18th and 19th centuries did (and we do in elementary algebra). For example the quantities represented by proportions in geometry (i.e. A:B::C:D) could be thought of as algebraic entities (therefore $A \cdot D = B \cdot C$, and so on). Fermat, building on the work of Vieta, had (by the late '20s or early '30s) the clear idea of analytic geometry:

> "Whenever in a final equation two unknown quantities are found, we have a locus, the extremity of one of these describing a line, straight or curved."

Indeed, so powerful was Fermat's study of analytic geometry and problems of curves, tangents and areas, that many of his results were central problems in the late 17th and 18th century discovery and development of the calculus. And just for completeness, you should note that even Cartesian coordinates are not, sadly, a Cartesian invention. Fermat was much more systematic about using orthogonal axes, but really it was Newton who crystallised the idea and use of *coordinate* systems.

In the **second** instance there is the case of Descartes' proposal of a mathematical physics. It ought to be noticed right away that Descartes was not being quite as original here as he might have left the impression he was: there had, of course, been "corpuscularians" and "atomists" before (although none so radical as Descartes), but more important, he was far from the first to apply mathematics to physics. Archimedes (c.250 B.C.), as readers of this journal know well, had beaten Descartes to it in his study of *the Equilibrium of Planes* (on the mathematics of statics - levers and balances, etc) and of

Floating Bodies (on hydrostatics). These results had been extended by later Greek scientists, most significantly by Heron (c.60 A.D.) and Pappus (c.320 A.D.) who used Archimedean like techniques to study simple machines like the balance, pulley, wheel, screw, or wedge. This "archimedean" study of mechanical phenomena had become particularly popular over the late 16th century in Italy and the Netherlands (where Descartes lived most of his adult life). So applying mathematics to physics (using mathematical analysis or mathematical models of simple mechanical phenomena to "explain" their behaviour) - to mechanics, moving bodies, fluid statics and hydraulics - was hardly new... it was almost commonplace.

However, there is more to this, and it is quite interesting because here Descartes was really very, very clear: he intended that his ***new*** physical science - which would solve all of the problems of physics in just a little while, without much bother (especially if he did it, of course..) - should be his new mathematical physics. He wanted physics to be sure & certain, and not open to attack by skeptics and other miscreants, and since mathematical knowlege was the most sure and certain kind of knowledge known, mathematising physics would be an excellent way of assuring that such knowledge would be really, really sure.

> In physics I should consider that I knew nothing if I were able to explain only how things might be, without demonstrating that they could not be otherwise. For, having reduced physics to mathematics, this is something possible, and I think I can do it within the small compass of my knowledge..."

Now, it is not entirely clear right away how this might work. Without loss of generality consider an arbitrary physical object: the piece of paper you are looking at. The Bad Old Science wouldn't think that a piece of paper had anything much that was "mathematical" about it: if you wanted to know about it, wanted to *explain* it, you'd want to know what caused it (by-and-large explanation means 'find the cause'), and there were four categories of causes: of what it is made, by whom - or how - it is made, what its shape or form is, and for what purpose it exists or what its use is or why it is. [If this doesn't sound like a causal explanation to you it is because of "differing criteria of explanative adequacy", as they call it in the trade.] That the paper might appear to have some mathematical or mathematisable properties - like the quantities of its dimensions and weight, for example, or its geometric properties like straight-line edges, flat, planar-like surfaces and 90° corners - these are all irrelevant and incidental to knowing about or being able to explain this piece of paper. After all, if it happens to weigh 70.01 g/m^2 or 70.02 g/m^2, in what way would this change the explanation of the causes of the paper? Such quantification is incidental to the nature of pieces of paper. And similarly, if you bend this piece of paper, its edges are no longer straight and its surfaces flat ... yet the piece of paper doesn't seem to have undergone any essential change.

Descartes doesn't bother to restrain his contempt for all this nonsense: to know about a piece of paper, you need to know how the corpuscles that comprise it act to give it its properties - its strength, colour, surface roughness, and so on. Fine. Terrific... but (1) how do you find these things out? (exercise left to the reader, of course), and (2) where does the maths bit come into it? Whatever a corpuscular explanation of this piece of paper might be, it is not entirely clear where all of this promised mathematical physics comes into it.

In fact if you happen to look through Descartes' scientific works (as opposed to the philosophy: you wouldn't expect to find much maths *there*), with the exception of his study of optics and refraction of light, it is noticeable how there really isn't any mathematics in it. Not only are equations not dense on the page, but an equation would positively die of loneliness. So either Descartes' constant promise to make physics mathematical was complete hogwash - which is possible, but not very impressive - or somehow we have missed the point. Clearly, since I have brought you this far, you must suspect the latter, and you would be right to do so.

Right at the beginning of his scientific work, it is just possible that Descartes' hope (that he was completely unable to fulfill) was that the micro-mechanical nature of phenomena (corpuscles in motion) could be mathematised (both matter and space being mathematisable quantities), and from such a mathematical micro-mechanics he would be able to derive a macro-mechanics. This seems to have been impossible, strangely enough! Later he was to try to work the other way, hoping that on the assumption of micro-mechanics, the macroscopic phenomena would enable him to infer the hidden microscopic mechanical phenomena:

> ...there are no rules in mechanics that do not hold also in physics, of which mechanics forms a part or species... Accordingly, just as those who apply themselves to the consideration of automata, when they know the use of some machine and see some of its parts, easily infer from these the manner in which others (which they have not seen) are made; so, from the perceptible effects and perceptible parts of natural bodies, I have endevoured to find out what are their imperceptible causes and particles underlying them."

Sadly, he couldn't work this either. Failed again.

But Descartes continued to maintain that his method would make physics just like mathematics, that he would mathematise physics and so make it as certain in its conclusions as the mathematicians were about theirs. Given the lack in his work of any appearance of what we (or anyone after the late 17th century physicists like Newton) might recognise to be mathematical physics, and his failure to mathematically establish a micro-macro mechanical link, you could be forgiven for thinking that his continued and firm claim to have mathematised physics is either blind over-enthusiastic optimism, or something worse. However, a careful reading of his method (*Discours de la methode*, 1637) reveals just what Descartes really meant by "mathematising physics": he meant to **borrow** from the mathematicians the method that he thought they used that makes their conclusions so sure and certain. You might note that in this, Descartes was not all that novel *per se*: other philosophers of science from Aristotle on have looked to see what it is that makes mathematics so certain, and tried to import that into the methods of science so as to make it certain.

In the *Discours* Descartes goes through a lot of careful philosophical argument about metaphysics and method, about epistemology and certainty, and so on and so forth, and central to this are what he calls his "four rules of method" for scientific procedure. If these rules are followed, he argues, they will not only lead you to an understanding of the underlying nature or causes of the phenomena studied, but will lead you to explanations of phenomena (or knowledge of them) that are true and certain... not merely probable knowledge, but evident and certain knowledge. The rules go:

(1) Never accept anything as true that is not known to be evidently so: that is to say, to carefully avoid precipitancy and prejudice, and to include in judgements nothing more that what presents itself so clearly and distinctly to the mind that there can be no occasion to place it in doubt.

(2) Divide each of the difficulties examined into as many parts as might be possible and necessary in order to best solve it.

(3) Conduct the investigation in an orderly way, beginning with the simplest objects and the easiest to know, in order to climb gradually, as by degrees, as far as the knowlege of the most complex, and even supposing some order among those objects which do not precede each other naturally.

(4) Make everywhere such complete enumerations and such general reviews as to make sure to have omitted nothing.

You may think that as rules of investigation or procedural rules for (all) science these would win prizes for mud-like opacity. And perhaps they ***do*** lack a certain immediate helpfulness for the practicing scientist... but do not despair just yet.

Remember two things. **Firstly** that Descartes was 'big' on algebra: applying algebra to geometry was, he thought, going to lead him to a mathematical "theory" or method that would unify all mathematics and provide the methods to solve all mathematical problems. **Secondly**, recall that one of the big names that Descartes would have known in the story of the extensions of Archimedean geometrical methods to the study or analysis of physical phenomena was Pappus; Pappus' work had only been translated and published and thought about in mathematical circles at the end of the 16th century, so it was relatively new and worth thinking about in Descartes' time (late 1610s & 1620s).

An issue amongst mathematicians at the time (1550-1650+) was what the discovery method of the ancient Greek geometers might have been: Greek mathematics, as it had come down to them, is rather like most mathematics as published today - formal, tight arguments in axiomatic or propositional form, with results stated and then proved. *How* the Greek mathematicians might have found out the results in the first place (their *heuristic*) was not known... and there was much speculation, more or less useful, about possible heuristic methods. However, the attentive reader of Book VII of Pappus' *Mathematical Collection* would have noticed that Pappus gives a clear hint as to what the method of the mathematicians was: it was the method of "analysis and synthesis". Pappus' description of what analysis and synthesis are is still about the most succinct:

> "***Analysis*** takes that which is sought as if it were admitted and passes from it through its successive consequences to something which is admitted as the result of synthesis: for in analysis we admit that which is sought as if it were already done and we inquire from what it is that this results, and again what is the antecedent cause of the latter, and so on, until by so retracing our steps we come upon something already known or belonging to the class of first principles.
>
> But in ***synthesis***, reversing the process, we take as already done that which was last arrived at in the analysis, and, by arranging in their natural order as consequences what before were antecedents, and successively connecting them one with another, we arrive finally at the construction of what was sought."

By the end of the 16th century there were a few mathematicians (Vieta and Hariot, for example) who noticed that the logical *procedure* of algebra is ***analytic***: you treat the unknown as if you knew what it was, i.e. "x", and then calculate until you find something that is known; so with $2x^2+3x=27$ you calculate away with x as if it were "something" until you find $x=3$, and now you know what x is. Indeed, Vieta called algebra the "analytic art". Descartes certainly thought he was on to an exceedingly good thing when he found how useful algebraic techniques were in finding how to solve geometric problems, so it was perhaps not a great leap for him to think that the lost, secret method of the Greeks for finding the solution to geometric problems was indeed analysis (some sort of algebra-like breaking down of a problem into component parts until these were reduced to solvable problems or knowns), followed by the synthetic proof by deduction from the axioms or things already proved, retracing the analytic procedure. If the analytic and synthetic method was the Greek's mathematical method and the method is so powerful at problem solving, perhaps **this** was the methodology science ought to adopt: not only would it be powerful and all encompassing, but it would lead (as in mathematics) to sure and certain (demonstrated) results. So perhaps it was in borrowing the mathematicians' analytic & synthetic method of procedure that Descartes proposed to "mathematise" physics.

Look at the rules of method again. Rule 1 is very typically Cartesian: doubt everything; when you begin the study of a phenomenon or a problem, get any previous explanations or preconceptions about it out of your mind. Sensible enough. Rules 2 & 3 are the core of the method: Rule 2 says that in order to understand a problem you must understand the first principles that govern it or underly it, or ultimately cause it; you get to them by breaking down the problem or phenomenon further and further until you find the irreducible, self-evident first principles (or ultimate laws). This, of course, is just analysis down to axiom-like first principles (which, because they are like axioms, will be self-evidently true). Here you can do such things as experiments so as to investigate the constituent parts of a phenomenon; this is permissible because whilst experiments don't lead you to certainty (fallibility of observations, etc.) they can help reveal the first principles.

Rule 3 is, naturally, synthesis: having uncovered the axiom-like first principles of a phenomenon, you now proceed by "deduction" (or ordered logical reasoning) <u>from</u> the clear, self-evident certain principles to explain phenomena, and from explanations of simple phenomena, on to explanations of more complicated phenomena: this chain of explanations would, obviously, be true and certain, not probable. We end up with a nice touch in rule 4, showing a mathematicians' sensitivity in Descartes: it says that at the end of this analytic and synthetic proceedure, check for completeness. In anachronistically modern terms, this sounds roughly like checking that your axiom base allows deductions of explanations of everything it <u>ought</u> to explain. If not, you have either got a degenerate set of axioms or you are missing some that ought to be there: not all of the causative first principles have been uncovered. There is, after all, no guarantee that your analytic procedure would have revealed **all** the "axioms".

So considered in this way, Descartes' programme for mathematising physics was, in a sense, to mathematise physics after all. It was not completely implausible for him to argue that such an analytic & synthetic method in science would make science "like mathematics" in that scientific study would proceed by a method analogous to the method he thought was the powerful and truth revealing heuristic and proof method of the mathematicians, and the use of such a method in physics would result (if everything went according to plan) in a demonstrated, necessarily true science. However, it should be noted that making physics "mathematical" by borrowing such an analytic & synthetic method doesn't seem to require or need to result in a mathematised physics in the modern (or Newtonian) sense of quantitative, mathematical models. You might even feel that such a methodology might not be all that far from those who nowadays seek to write physical theories in axiomatic form - axiomatic Quantum Mechanics or General Relativity and so on.

As a final point, it is worth remembering that the use of the analytic and synthetic method in physics was to be seen by other methodological reformers of the 17th century as a fruitful way of linking heuristic, experimental methods to methods of proving the truth of physical theories. In particular, Newton (who spent a fair amount of his spare time in later life abusing Descartes and the Cartesians, and whose methodology is often held to have been a Good Thing) can be seen as presenting his results and his theories in a quasi-axiomatic, propositional **synthetic** geometric form, having arrived at his results by experimental and mathematical **analysis**. As he said in his *Opticks*:

> "As in mathematics, so in natural philosophy [i.e. science] the investigation of difficult things by the method of analysis ought ever to precede the method of composition [synthesis]. This analysis consists in making experiments and observations... [and so] proceed from compounds to ingredients... and in general from effects to their causes, and from particular causes to more general ones till the argument end in the most general. This is the method of analysis: and the synthesis consists in assuming the causes discovered and established as principles, and by them explaining the phenomena proceeding from them, and proving the explanations."

So perhaps Descartes wasn't really such a clod... and even if it can hardly be said that the promises of the method and of (the rhetoric of) a "physico-mathematics" are ever fulfilled, he was at least thinking along lines for scientific method that - very soon - others were also to see as the way of the new science. A mathematico-physics can be mathematical physics without being Mathematical Physics after all.

Why 12 x 2 = 25

Ian White

The hardest thing about going to school in Banjul is often just getting there. Banjul is a very small capital city enclosed by mangrove swamps, so half of our students and almost all the teachers live outside the capital and travel 10km across the mangrove to school each day. If you are lucky you can get a lift in a private car, but the majority have to struggle to get into minibuses or take long round-about bus journeys.

When I walk into class at 8.20 to take the register, the students scramble to find cloths and meticulously wipe the dust off their desks and chairs. A lot of dust is brought in from the Sahara by the Hamattan wind. On some days it is so heavy that they need to dust again after break-time.

A full class consists of fifty students. It sounds a very large number but I became used to it very quickly. Now I only realise how large the class is when everyone hands their book in. With classes of fifty it is difficult to find time for individual teaching and most of the time it's chalk and talk. The students put up with unimaginative teaching amazingly well, but they do appreciate novelties and practical demonstrations, like using trigonometry to measure the height of the chapel's bell tower.

The commonest method for the students to study is by rote-learning, which may work well for history or biology but is a disaster for maths. The students see maths completely divorced from real life. Perhaps this is not surprising, since the local culture contains very little conventional maths. In the market, for example, foodstuffs are sold by the pile; each pile costs a round sum like 25 bututs or 1 dalasi. Most people identify sums of money by the actual coins not by abstract numbers. The names of the coins are left-overs from the days of British rule: a 5 Dalasi note is colloquially called a pound, 25 bututs is a shilling. A cup of groundnuts (peanuts) costs "sixpence", which means 12 bututs - and so it works out that two cups of groundnuts cost 25 bututs. Fortunately the anomalous butut is worth so little that it is not worth fighting over.

At my school we teach modern maths up to form 3, using textbooks written in Ghana. After form 3 only those students in the science stream (who are presumably the most able at maths) continue with modern maths; the others switch to commercial maths. This has a more useful syllabus, but the change of style makes it very hard for most students. So while modern maths scores excellent results in O level, commercial maths results are very poor and discouraging.

At eleven we break for half an hour. Women come in to the school compound and sit under the casuarina trees selling sandwiches, groundnuts, fish pies, sweets, ices and ginger beer; the well-to-do buy Coke outside the staff room. The students chat in half a dozen local languages and play football on the basketball court, and tennis on the volleyball court.

Then we go back into classes until 2 pm. All the familiar conflicts happen here too, of course. There are students who don't hand in homework or who copy in tests. Tempers can rise high and if someone insults someone's mother you know you are probably going to have a fight on your hands. Black skin is still white underneath when stabbed with a pair of compasses, and blood is still red.

Communication with parents is very difficult. The Gambian postal system covers only those people who can afford Post Office Boxes, so the only way the school can send out reports is by entrusting them to the students themselves. This is unreliable, of course. Some students get consistently bad reports and their parents are never even aware that reports exist.

At 2 o'clock then, classes finish; except on Fridays when they finish at one, to allow Muslims to get to the Mosque for their weekly prayers. For students living outside Banjul, it's back to the roadside to wait for a lift home. Students who live in Banjul are expected to return to school after lunch for private studies. During this time there is such quiet, as if nobody were in school at all; perhaps, one wonders, education is more successful in the absense of teachers?

Life also has its surprises. Muslim holidays depend on sightings of the moon so they are never completely predictable. Often we discover about a holiday with only a day or two's notice; once we only found out as we were teaching the second lesson on the holiday itself. At other times I find half-empty classes because those who haven't paid their school fees (£7 a term) have been suspended. At first I strongly disliked this approach to collecting fees but, having seen how quickly they are paid once suspensions are enforced, I realise that a number of students just want to see if they can avoid parting with their father's money.

Visits by two English school parties entertained us this year. Both times they played our school at basketball in front of a large crowd, and were crushingly defeated. Afterwards my students joyfully greet me with unsound generalisations about British people's sporting abilities.

When I leave school there is time to go for a swim in the sea or for a walk. Often I bump into my students, which is a pleasure as they are generally both friendly and respectful. When I travel in the rural areas, I may meet one of my students, who takes me to greet his father. I try to expess in my basic Mandinka language how well the son is doing, and the father makes me a small gift like a few oranges. Every time, I find that these students from rural areas are the best behaved and the most conscientous in class. But such travels are for the holidays. On a normal day I am content to go home, eat a large bowl of rice and groundnut stew and fall asleep.

Modelling using the Poisson Distribution

D.J.Colwell and J.R.Gillett

Goal scoring in football is often given as an example of a situation which can be modelled by a Poisson distribution. To test this claim for a set of data giving the number of goals scored in matches, the chi-squared distribution is used to investigate the goodness of fit of the model. Hence the claim usually takes the form of stating that, at a certain level of significance, there is no evidence for rejecting the hypothesis that goal scoring is modelled by a Poisson distribution, with a mean equal to the mean number of goals scored per match.

Of course this is not a proof of the suitability of the model. However, further evidence for the suitability of the Poisson model may be obtained by considering the time intervals between goals.

Suppose that events in a Poisson process are occurring at a rate λ (>0) per unit interval of time. Then the random variable representing the interval between successive events has an exponential distribution with parameter λ. More generally, the random variable representing the length of the interval between an event and the next but n-1 events ($n \geqslant 2$) has a gamma distribution with parameters n and λ.

We have followed up these ideas by considering the set of data provided by the goals scored by the losing team in each of the F.A. Cup Finals since 1885. Complete data for the times at which the goals were scored does not seem to be readily available for the Cup Finals held before 1885.

We have used data for the losing teams only, since winning teams will, by definition, eventually score at least one goal. Hence the goals scored by the winning teams cannot be modelled by a Poisson distribution.

Some of the finals required extra time, extending the usual 90 minutes of playing time to 120 minutes, while others required a replay. Including these replayed matches we have analysed data from 99 matches, 13 of which required extra time.

On fitting Poisson distributions to the data we obtain the following results:

90 minute matches. (Mean number of goals scored by losing team /match = 0.5992)

Number of goals scored by the losing team	0	1	2	3	>4
Observed frequency	48	28	9	1	0
Expected Frequency	47.23	28.30	8.48	1.69	0.30

Using 3 groupings we obtain a chi-square value of 0.02, with 1 degree of freedom. [$\chi^2_{0.10}(1) = 0.016$, $\chi^2_{0.25}(1) = 0.102$]

120 minute matches. (mean number of goals scored by losing team /match = 0.7989)

Number of goals scored by the losing team	0	1	2	3	>4
Observed Frequency	5	6	2	0	0
Expected Frequency	5.85	4.67	1.87	0.50	0.11

The small amount of data here does not allow a chi-square test to be used. However, inspection offers no suggestion of a poor fit.

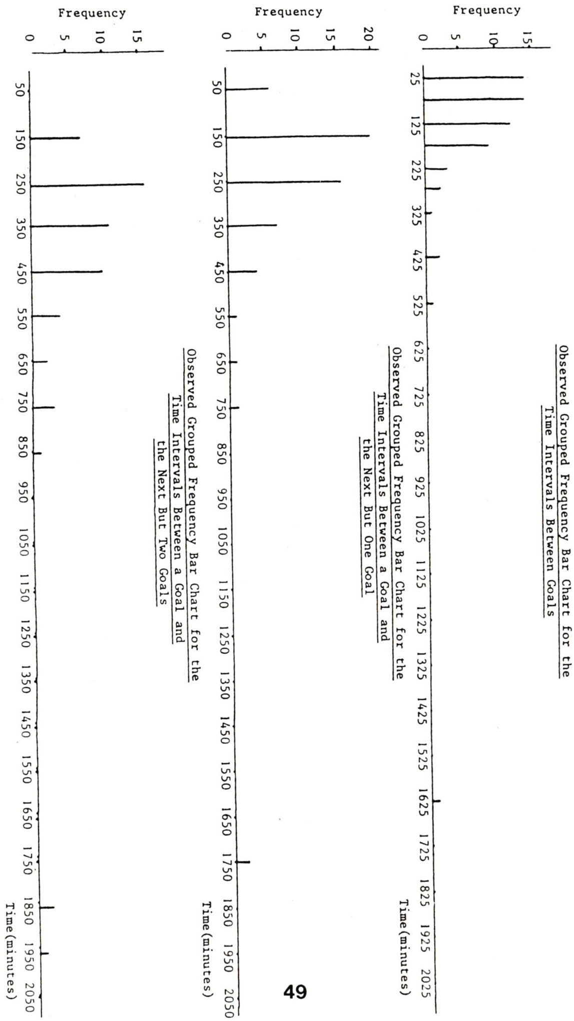
The time intervals (in minutes) between goals scored by losing finalists since 1885 are given below.

412, 178, 110, 287, 101, 110, 7, 86, 82, 167, 42, 78, 138, 72, 418, 35, 144, 1608, 178, 48, 192, 84, 51, 146, 237, 21, 166, 31, 94, 315, 38, 15, 57, 29, 83, 61, 158, 159, 258, 120, 20, 30, 151, 23, 51, 119, 235, 63, 61, 147, 530, 214, 2, 120, 100, 45, 150, 108, 74.

[The mean time interval $\frac{1}{\lambda}$ between goals is 150.19 minutes.]

Grouped frequency distributions associated with this data are illustrated in the following diagrams. These display the characteristics of the appropriate exponential and gamma distributions and this can readily be confirmed by testing their goodness of fit using chi-squared tests.

The quality of the goodness of fit is perhaps surprising when some of the characteristics associated with the data are considered. For example the matches considered, being spread over a hundred years, have been played between many different teams, playing on many different types of ground condition and with greatly varying styles of play. In spite of this we have found no evidence for rejecting the Poisson model for goal scoring nor, indeed, the appropriate exponential or gamma distribution models for the time intervals between goals.



Observed Grouped Frequency Bar Chart for the Time Intervals Between Goals

Observed Grouped Frequency Bar Chart for the Time Intervals Between a Goal and the Next But One Goal

Observed Grouped Frequency Bar Chart for the Time Intervals Between a Goal and the Next But Two Goals

Conference

Douglas Quadling

There are a number of societies in this country concerned with mathematics. Some of them are "learned societies", and are academically very prestigious - such as the London Mathematical Society. Others are associations of people who practice mathematics in particular ways. The Mathematical Association, to which the Archimedeans have been affiliated for many years, is primarily (but by no means exclusively) for people interested in **teaching** mathematics - whether in schools, universities, or further education. It is well known for its journals, the *Mathematical Gazette* and *Mathematics in School*, and for its reports, which cover every conceivable aspect of mathematics teaching. Issues with which the Association is especially concerned at present are ways of using the micro in teaching mathematics, and new approaches to the assessment of mathematical achievement.

The Mathematical association holds a conference each year around Easter time. The venue moves around the country - recent years have seen it in Swansea, Exeter, Manchester and Dundee - and in 1986 it is to be in Cambridge. The Archimedeans are represented on the conference committee by David Jessop, and a special rock-bottom conference fee of £2 has been arranged for members who would like to attend and can arrange their own accommodation; colleges are being asked to be sympathetic to students who want to occupy their rooms during the conference period, which is Wednesday to Saturday, 2-5 April. In return, the Association would be grateful for help at the conference, such as guides to the Corridor (the venue being Newnham and Selwyn) and help with registration on the Wednesday. There is also the Association's Annual Dinner, which will cost a further £12.00 for those who want it. There will be a special application form for members of the Society, which the Society will be circulating early in the Lent term.

Speakers will include Prof. Sir Hermann Bondi, Prof. Shephard of U.E.A. (on "tiling") and Tony Fitzgerald of Birmingham (on "uses of mathematics in industry"); in addition, there will be "working groups" on a wide range of topics. This year's President is Miss Hilary Shuard, deputy principal of Homerton College.

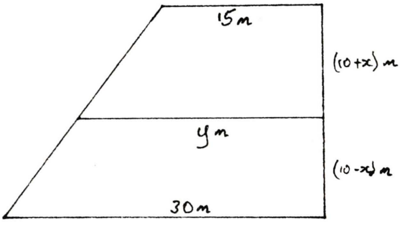
We look forward to seeing many of you at the conference, and hope that it will strengthen the links between the Archimedeans and the Mathematical Association.

Answers to the Problems Drive

Paul Smith

1) Answer : 404
Sequence: 29, 46, 38, 34, 26, 22, 25, 27, 21, 28, 16, 18, 24, 20, 30.

2) Answer : $10(\sqrt{10} - 3)$m

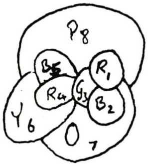


$y = 15(\frac{1}{20}(10 + x) + 1) = \frac{3}{4}(30 + x)$
$(15+y)(10+x) = (30+y)(10-x)$
Therefore $x^2 + 60x - 100 = 0$

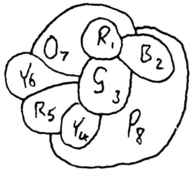
3) Use numbers for letters :
i) 22 , 7
The sequence was the number for the third letter in each of ONE, TWO, THREE...
ii) 65 , 49
The sequence was the sums of values in ONE,TWO,THREE...
iii) 17 , 15
The sequence consisted of (maximum letter value) - (minimum letter value)
iv) 22 , 23
The vowels in ONE, TWO, THREE...
v) 177 , 395
$(\text{last letter})^2 - (\text{first letter})$

4) Answer : 8

eg.



or



5)
J,R girls
I,S boys

S gave g to J and received h from her .

S	e	R	a
I	d	J	b
I	f	R	c

6) There are various solutions
eg. L L L R L R R R L L L R R L L R L R L L R R L

7) Answer : 2,899,078

With the white piece off all symmetry axes :

$$\tfrac{1}{2}(\ 36 \times 360 \times 359\ ) = 2{,}326{,}320$$

With the white pieces on just one axis :
i) blacks symmetric wrt this axis

$$18 \times (\tfrac{1}{2}(18 \times 17) + 342/2) = 5{,}832$$

ii) blacks not symmetric

$$18 \times \tfrac{1}{2}(\tfrac{1}{2}(360 \times 359) - (\tfrac{1}{2}(18 \times 17) + (342/2))) = 578{,}664$$

White piece central , both blacks on axis :

$$18 \times \tfrac{1}{2}(44 + 2) = 414$$

White piece central , one black on axis :

$$18 \times 144 = 2{,}592$$

White piece central , no blacks on axis :

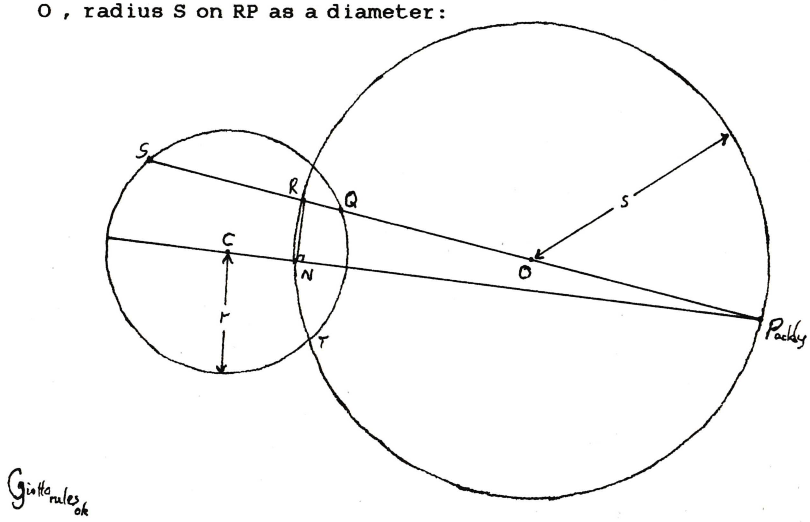
$$\tfrac{1}{2}(36 \times (287 + 5)) = 5{,}256$$

Total number of positions = 2,899,078

8) Answer : 14
eg BBBB -> BBBF -> BBFF -> BFFF -> FFFF -> FFFB -> FBFB -> FBBB -> FBBF -> FFBF -> BFBF -> BFBB -> BFFB -> BBFB .

9) ANSWER : 7
If n = 1,2 #3 is not needed.
If n = 3 #3 is not needed after 8pm Friday
If n = 4 #3 is not needed after 2am Saturday
If n = 5 #3 is not needed after 5am Saturday
If n = 6 #3 is not needed after 2pm Saturday
If n = 7 #3 is not needed after 6pm Saturday

10) The fence is a straight line joining the two points where tangents from P meet the field. To see this draw a circle centre O , radius S on RP as a diameter:



We are given $1 = (s-OQ)(2s+RS)/RS(s+OQ)$. So $RS.OQ + s.OQ = s^2$ ie $s^2 = OQ.OS$ So OT is a tangent to the field (Intersecting Chords Theorem) hence CT is a tangent to the other circle, so $CT^2 = CN.CP$, hence CN is fixed as required.

11)

7	1	8	9	0	5	7
1	X	8	9	X	3	1
3	X	6	X	X	X	8
X	X	9	7	X	2	9
3	3	7	5	0	0	0
9	3	4	X	X	X	5
2	0	3	X	3	1	7

To begin with, see that by the size of 1a, 14a must begin with a 2 (by 1a) But 1a starts and ends with the same digit ,therefore 14a must be 203. Given that the rest is trivial.

12)
i) 79 , 89
Omit numbers containing 2,3,5 or divisible by 2,3,5.
ii) 33 , 24
From Roman numerals : 9 -> IX -> I + X -> 9 + 24 = 33
iii) 25341 , 231689

nth term is $A_n = 1 + n \sum_{1}^{n-1} A_i$

iv) 1081 , 1219
Numbers except 23 which have 23 as the lowest prime divisor.
v) 102.53 , 52.66
Geoff Boycott's 1^{st} class batting averages in '72 , '73 ...
(Well,what else could the title refer to ?)

Loose Ends

"Archimedean & Platonic Solids" is not the only publication to have been sent to us in the last year. The one other book, "Basic Technical Mathematics" by Stuart R. Porter and John F. Ernst (Addison-Wesley, £31.95, September 1985, hardback, c.800 pages), asked for a review. I have no desire to enter into the world of book reviews, unless a specific book truly deserves it. I will restrict myself to saying that this book appears, at the cursory glance I have given it, to be of roughly A level standard and not of outstanding relevance to undergraduates. Anyone wishing to peruse it, or having a suggestion for some worthwhile place to give it to, please see the Editor.

Beside the two books, we have recieved "Boletim Da Sociedade Brasileira Di Matemática", "Annales Universitatis Scientiarum Budapestinensis De Rolando Eotvos Nominatae, Sectio

Mathematica" and "Acta Mathematica Hungarica" for which I am most grateful; I also have various issues of several other journals, which I think predate my editorship. Anyone wishing to peruse this collection should, as ever, contact the editor, i.e. me.

The Archimedeans do now have an office in the basement of DPMMS (aka Seminar room 4), for which we are most grateful to the department. Please would people use the office purely as an office, not as a place for gathering large crowds or a place of hustle and bustle. The department is a place of research, and we should not disturb it. That said, legitimate use of it as an office is fine. Notably, if officers and agents wish to keep things in the office when they are away over Easter, that is fine. Equally, any ex-officers, ex-agents or other trouble-makers who have Archimedeans junk, cluttering up their rooms, which they somehow never got round to finding who to pass on to, now is your chance to return it. Please try to keep the place tidy. Above all else, remember that we have the office more or less 'for a trial period' out of the department's kindness, not out of any bizarre 'divine right' to an office.

I am further endebted to the department for the use of their word-processor and printer, to the Computing Service for the use of their text-processing facility, and to a wide variety of people, particularly Paul, for help with the complexities of using all this high-tech machinery. Thanks also to Bob for typing various things, to Prof. Conway for help on his article, to Piers for typing his, to John Hunton for the covers, to Nigel for his work and to Piggots the printers.

Any faults in this issue should be blamed on me, as Editor, Referee, Typist, Proof-reader, Compositor or Illustrator, as appropriate.


Eureka
46
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