Eureka Editor
archim-eureka@srcf.net
The Archimedeans
Centre for Mathematical Sciences
Wilberforce Road
Cambridge CB3 0WA
United Kingdom

Published by The Archimedeans, the mathematics student society of the University of Cambridge

Thanks to the Betty & Gordon Moore Library, Cambridge

EUREKA

THE ARCHIMEDEANS' JOURNAL

NUMBER 38

1975/76 PRICE 30p

EUREKA

This issue of Eureka costs 30p. It is sent free to all undergraduate mebers of the Archimedeans. Individual copies will be supplied for 40p each by post, but a subscription may be taken out, for which subsequent copies will be charged at the cover price in force at the time when the subscription was last paid - for example, a sum of £1.50 will entitle a subscriber to the next 5 copies.

Enquiries about subscription, contributions, etc. should be sent to

"Eureka"
The Arts School
Bene't Street
Cambridge

Cheques, etc. should be made payable to
"The Archimedeans : Eureka"

"Car moi, je ne crois pas à la mathématique"

A. Einstein

EUREKA

Number 38

Editor Richard Pinch

Business Manager Geoff Chapman

Contents

EDITORIAL

In this copy of 'Eureka' we have concentrated on the general theme of "problems", ranging from the amusing to the unsolved. (Indeed, when Prof. Erdos uses the word "difficult" this is very much of an understatement!) This tendency of mathematicians to divert themselves with puzzles and problems of a recreational nature is an indication of one of the major patterns of expansion in the subject - we might almost say that modern algebraic number theory grew out of the famous Last Problem of Fermat, or modern graph theory out of the Four-Colour Problem.

But we must not underestimate a contrary tendency, for synthesis and unification. The elegant work of Bourbaki has lead to advances which cannot be associated with any well-known or historic problems as such, although the clarity of their work has made major contributions to the progress of the subject.

It must be borne in mind that the methods used to solve any problem, whether posed by Hilbert or by a fellow--undergraduate, are capable only of bearing their full fruit when examined apart from the use to which they have been put; and the second proof of an outstanding problem is usually better than the first.

Having said which, it remains only for us to welcome our readers into the pages of this issue and to hope that they have as much enjoyment in considering some of our problems as our contributors have had in posing them.

CONSECUTIVE INTEGERS

by P. Erdos

Very recently two old problems on consecutive integers were settled. Catalan conjectured that 8 and 9 are the only consecutive powers. First of all observe that four consecutive integers cannot all be powers since one of them is congruent to 2 modulo 4. It is considerably more difficult to prove that three consecutive integers can not all be powers; this was accomplished about twenty years ago by Cassels and Makowski. Finally in 1974 using some deep results of Baker, Tijdeman proved that there is an n_0, whose value can be given explicitly, such that for $n > n_0$ n and $n+1$ are not both powers. This settles Catalan's conjecture nearly completely, and there is little doubt that it will be settled in full soon. It has been conjectured that if $x_1 < x_2 < x_3 \ldots$ is a sequence of consecutive powers, $x_1 = 1, x_2 = 4, \ldots$ then $x_{i+1} - x_i > i^c$ for all i and some absolute constant c. At the moment this seems intractable. (The paper of Tijdeman will appear in Acta Arithmetica.)

It was conjectured more than a century ago that the product of consecutive integers is never a power. Almost 40 years ago, Rigge and I proved that the product of consecutive integers is never a square, and recently Selfridge and I proved the general conjecture. In fact, our result is, that for every k and l there is a prime $p \geq k$ so that if

$$p^{\alpha_{k,l}} \,||\, \Pi_1^k (n+i)$$

then

$$\alpha_{k,l} \not\equiv \bmod.(l).$$

We conjecture that in fact for all $k > 2$ there is a prime $p \geq k$ with $\alpha_{k,l} \equiv 1$, but this is also intractable at the moment.

It often happens in number theory that every new result suggests many new questions - which is a good thing as it ensures that the supply of Mathematics is inexhaustible! I would now turn to discuss a few more problems and results on consecutive integers and in particular a simple conjecture of mine which is more than 25 years old.

Put

$$m = a_k(m)b_k(m),$$

$$a_k(m) = \Pi p^{\alpha_p}$$

where the product extends over all the primes $p \geq k$ and $p^\alpha \,||\, m$. Further define

$$f(n;k,l) = \min\{a_k(n+i) \mid 1 \leq i \leq l\}$$

$$F(k,l) = \max\{f(n;k,l) \mid 1 \leq n \leq \infty\}.$$

I conjectured that

1) $$\lim_{k\to\infty} F(k,k)/k = 0$$

In other words, is it true that for every ε there is a k_ε such that for every $k \geq k_\varepsilon$ at least one of the integers $a_1(n+i)$, $i=1,\dots,l$, is less than k_ε. I am unable to prove this but will outline the proof of

2) $$F(k,k) < (1+\varepsilon)k \text{ for } k > k_0(\varepsilon).$$

To prove (2) consider

3) $$A(n,k) = \tilde{\Pi}_1^k a_1(n+i)$$

where in (3) the tilde indicates that for every $p \leq k$ we omit one of the integers $n+i$ divisible by a maximal power of p. Then the product $\tilde{\Pi} a_k(n+i)$ has at least $k-\pi(k)$ factors and by a simple appilication of the Legendre formula for the factorisation of $k!$ we obtain

4) $$\tilde{\Pi} a_k(n+i) \mid k!\ .$$

If (2) did not hold, we have from (4) and Stirling's formaul

5) $$((1+\varepsilon)k)^{k-\pi(k)} < k^{k+1}\exp(-k)$$

or $$k^{\pi(k)+1} > \exp(k)(1+\varepsilon)^{k-\pi(k)}$$

Now, by the prime number theorem,

$$\pi(k) < \frac{(1+\varepsilon/10)k}{\log k}$$

and so from (5),

$$k\uparrow\left((1+\varepsilon/10)\frac{k}{\log k}+1\right) >$$
$$> \exp(k).(1+\varepsilon)\uparrow\left(k-\frac{2k}{\log k}\right)$$

which is false if k is large enough, and this contradiction proves (2).

Assume for the moment that (1) has been proved. Then one can immediately ask for the true order of magnitude of F(k,k). I expect that it is $o(k^{\varepsilon})$ for every $\varepsilon>0$. On the other hand, I can prove that

6) $$F(k,k) > \exp\left(c.\frac{\log(k)\log\log\log(k)}{\log\log(k)}\right)$$

The problem of estimating F(k,k) and the proof of (6) is connected with the following question on the seive of Eratosthenes-Prim-Selberg : determine or estimate the sm smallest integer A(k) so that one can find, for every p with $A(k) \leq p \leq k$, a residue u_p such that for every integer $t \leq k$, t satisfies one of the congruences to u_p modulo p. Clearly $F(k,k) \nmid A(k)$. Using the method of Rankin-Chen and myself I proved

7) $$A(k) > \exp(c.\log(k)\log\log\log(k)/\log(k))$$

which implies 6. I do not give the proofs here. It would be interesting and useful to prove $A(k) < k^{\varepsilon}$ for every $\varepsilon>0$ and sufficiently large k.

Now, I shall say a few words about F(k,l) for $k \neq 1$. It follows easily from the Chinese Remainder Theorem that for $l \leq \pi(k)$ we have $F(k,l) = \infty$, since for a suitable n, we can make $n+i$, $1 \leq i \leq \pi(k)$ divisible by an arbitrarily large power of p_l. It is easy to see that this no longer holds for $l = \pi(k)+1$ and in fact it is not hard to prove that

$$F(k,\pi(k)+1) = \prod p^{\alpha_p}$$

where $$p^{\alpha_p} \leq \pi(k) < p^{\alpha_p+1} .$$

As l increases it gets much harder to even estimate $F(k,l)$. Many more problems can be formaulated which I leave to the reader and only state one which is quite fundamental:

Determine or estimate the least $l = l_k$ so that $F(k,l_k)=1$.

In other words, the least l_k so that among l_k consecutive integers there is always one relatively prime to the primes less than k. This question is of course connected with the problem of estimating the differnce of consecutive primes and also with the following problem of Jacobsthal: Denote by $g(m)$ the least integer so that any set of $g(m)$ consecutive integers contains one which is relatively prime to m. At the recent meeting on Number Theory in Oberwolfach (Nov.'75) Kanold gave an interesting talk on $g(m)$ and the paper will appear soon. Vaughan observed that the seive of Rosser gives $g(m) < (\log(m))\uparrow(2+\varepsilon)$ for all $\varepsilon > 0$ if m is sufficiently large. The true order of magnitude is not known.

It seems to me that interesting and difficult problems remain for $l \leq \pi(k)$ too. Here we have to consider the dependence on n too. It is not hard to show that for every $\varepsilon>0$ there are infinitley many values of n for which

8) $$f(n;k,l) > (1-\varepsilon)^{l}\sqrt{n} .$$

The proof of (8) uses some elemtry facts of Diophantine approximation and the Chinese Remainder Theorem. We do not give the details. I do not know how much (8) can be improved. By a deep theorem of Mahler, using the p-adic Thue-Siegel Theorem, $f(n;k,l) > n^{(\varepsilon+1/l)}$. It is quite possible that

9) $$\limsup_{n\to\infty} f(n;k,l)^{l}/n = \infty.$$

Interesting problems can also be raised if k tends to infinity with n; e.g. how large can $f(n;k,\pi(k))$ become if $k = (1+o(1))\log(n)$? It seems to be difficult to write a really short note on the subject since new problems occur while one is writing!

It would be of some interest to know how many of the integers $a_k(n+i)$ must be different. I expect that more than c.k are. If this is proved one of course must determine the best possible value of c.

Denote by K(l) the greatest integer below l composed entirely of primes below k. Trivially

10) $$\min_{n} \max_{i} a_k(n+i) = K(l)$$

To prove (10) observe that on the one hand any set of l consecutive integers contains a multiple of K(l), on the other that if 2l divides t, then the integers $t!+1,\dots,t!+l$ clearly satisfy (10), when n=0. More generally, try to characterise the set of n which satisy (10). To simplify matters, let k=l and denote n_k as the smallest positive integer with $\max_i a_k(n+i) = k$, S_k as the class of all integers n such that this is true. If p^{a_p} is the greatest power of p not exceeding k then

$$\prod_{p\le k} p^{a_p+1} \in S_k .$$

Perhaps I am overlooking an obvious explicit construction for n_k but at the moment I do not even have good upper or lower bounds for it. When is k! in S_k , The smallest such k is 8 and I do not know if there are infinitely many such k's. By Wilson's theorem, p! is never in S_p.

To complete this note, I state three more extremal problems in number theory. Put

$$n! = \Pi a_i, \quad a_1 \le a_2 \le a_3 \ldots \le a_n.$$

Determine $\max\{a_1\}$. It follows easily from Stirling's formula that a_1 does not exceed $(n/e)(1-c/\log(n))$. I conjectured that for every $\eta>0$ and suffciently large n, $\max a_1$ exceeds $(1-\eta)n/e$.

Put

$$n! = \Pi b_i, \quad 1<b_1<b_2<\ldots<b_k\le n$$

Determine or estimate min k .

Clearly k exceeds $n - n/\log(n)$ and by more complicated methods I can prove

$$k = n - (1+o(1))n/\log(n)$$

$$k > n - n(\log(n)+c)/(\log(n))^2$$

where c is a positive absolute constant.

Put

11) $$n! = \Pi u_i, \quad u_1 u_2<\ldots<u_k$$

Determine or estimate $\min u_k$ - k is not fixed. It is not hard to prove that u_k less than 2n has only a finite number of solutions. I only know of two:

$$6! = 8.9.10$$

and $$14! = 16.21.22.24.25.26.27.28 .$$

It would be difficult to deterimne all the solutions, although Vaughan has just found some more -

$$3! = 6$$

$$8! = 12.14.15.16$$

$$11! = 15.16.18.20.21.22$$

$$15! = 16.18.20.21.22.25.26.27.28$$

and this is all up to 15. Vaughan also tells me

$$40! = 42.44.45.48.49.50.51.52.54.55.56.57.$$

$$.58.59.60.62.63.64.65.66.68.69.72.74.80$$

THE ARCHIMEDEANS

by H.G. Stark

Whether the titles suggested, inspired the speakers or vice-versa, the lunchtime talks were well-attended and well worth attending this year. Prof, Lighthill gave a very animated talk (with slides) about the 'Aerodynamics of Flight' and continuing the biological theme Dr. ffitch explained 'Camal-driving through a net' and presented a CAMAL manual to the Society, for the minutes. It was therefore thought by some mebers that Prof. Adams, answering the question 'Is the theory of games any use to the Go-player' (in the negative) should have also donated his gigantic Go-board. Dr. Miles Reid gave an unbelievable performance when tackling the question 'Do Algebraic Curves exist' and a very interesting explanation of 'Regges pole - Barren wilderness to capit alistic exploitation' was given by Prof. Loretta Jones of the University of Illinois.

The evening meetings of the Lent Term began with Sir Morien Morgan, Master of Downing, recalling his experiences at Farnborough and in the Air Ministry. The Society learnt about the Otmoor Project, and discussed whether 'Seeing is Believing' later in the Term.

The outcome of the trip to Oxford was a draw, but there could be no doubt that the Adams Society of St. John's soundly thrashed the Trinity Mathematical Society (or possibly there could?), ,in a punt-jousting contest held at the beginning of the Easter Term. Cambridge won the Problems Drive against the Invariants from Oxford, held in Trinity Old Combination Room, and the wooden spoons returned to their traditional habitat.

At the punt party, rain and hail stopped play, that is, an ExtraOrdinary General Meeting and the traditional aquatic and arborial pursuits, and the company returned wetter than if the latter had taken place. However, there were no hitches on the croquet afternoon and the ramble the following day began at Staplford and continued through Newton, Foxton, Harston, Harlton and Grantchester. All but three of the party managed to cover exactly the same route, which must set a precedent.

The success of these ventures was due to the hard work of the Committee members, and with the new Committee I look forward to another good year for the Society.

ALPHAMETIC PROBLEMS

by J.R. Partington

All the letters represent distinct digits, and the solutions are unique.

1.
```
  APPLE
  APPLE
  HELPS
 ------
 NEWTON
```

2.
```
 EUREKA
 EUREKA
     AN
-------
 ANSWER
```

3. DO × LIKE = EUREKA

4. ALL × BEST = EUREKA

5. ALL × GOOD = EUREKA

9.
```
     ALL
     SAY
     ---
    ....
   ....
  ....
  ------
  EUREKA
```

AMAZING!

by E.J. Anderson

A maze is a connected finite graph (of nodes joined by lines) with two special nodes; the entrance and the exit. We thread the maze by finding a path from entrance to exit, starting at the entrance and at each node choosing the next line to take by some rule or strategy. Various strategies could be employed (e.g. at each node choose a line randomly, according each equal probability.) Usually we are only interested in strategies which use no information not available to someone threading the maze, i.e. the path taken so far and the number of lines from each node on it.

It is natural to ask what, subject to this constraint, is the best strategy to use, in the sense of finding the quckest path to the exit. Unfortunately, this seems an impossible problem, so we shall ignore it and think instead about a particular, very reasonable rule - the "normal strategy". This may be expressed as follows:

> 'At any node choose an unused line if there is one, otherwise go back along that line, not traversed twice, which has been used most recently.'

This has several interesting properties, not least of which is the result we shall lead up to. First we have a simple

Lemma

Using the normal strategy, if a path visits a line twice, than at the time of the second visit each line visited in between will have been visited twice exactly, and there will be no unused lines from any of the nodes visited in between.

Proof:

Consider the first time the path doubles up. This must be when it turns immediately back on itself. After this it will retrace its steps until branching off down the first untried line. Having done this, the doubly-traversed part of the maze is now inaccessible, and we may remove it. We may now continue inductively, removing sections at each stage, with the statement of the lemma reamining true. Hence the result.

The normal strategy can now be seen to be well-defined. At any node other than the entrance the lines to that node will have been used an odd number of times, so there will be aline which has bot been used twice, and we can choose the next line in accordance with the strategy. However, if each line at the entrance has been used twice, then, by the Lemma, since the maze is connected, the exit must have been visited.

Next we assign to each line in a maze a length and make the following definitions:

The length of a path to be the sum of the lengths of its constituent lines.

A route to be a line-disjoint path from entrance to exit.

A simple maze to be one with only one route.

Then we have the following theorem:

Theorem

For any simple maze M, let L be a random variable giving the length of a path from entrance to exit in M and S(M) the sum of the lengths of all the lines in M.

Then

$$E(L) = S(M) .$$

Proof:

Let R be the route in M, and C_i, i=1,...,N the components of M-R. As M is simple, each C_i meets R once only.

Now, from the Lemma, we see that if any line of C_i is traversed then the whole of C_i is traversed exactly twice. But at each node of R we choose the next line randomly, and so

$$\begin{aligned} &\text{prob}\{C_i \text{ entered}\} = \\ &\text{prob}\{C_i \text{ entered before correct line}\} \\ &= \tfrac{1}{2} \end{aligned}$$

The lines of R are bound to be traversed. QED

To allow us to extend this result, we class mazes into two types. We call a maze M of class A if for every route R of M and component C_i of M-R, the last node of R which meets lines of C_i meets only one such line - otherwise, call it class B.

For example, of the mazes in Fig.1, (1) and (2) are of class A, (3) and (4) of class B. Note the directional nature of this definition.

With each path through a maze we can associate a route by removing lines used more than once; this would not be so easy if we were not using the normal strategy. It is clear that a path is associated with a route R if each time that it leaves R at some node it later returns to R at that node having traversed all the intermediate lines just twice. This can happen only if the component of M-R which it enters is only incident with R at nodes before this one, and only once there. This motivated our definition of class A, for in this class the paths associated with a particular route R cover the whole maze and consist of all paths in the simple maze M' where each component of M-R is joined to R only at the last possible node. E.g. Fig.2 shows M' for M the maze (2) in Fig.1.

We are now ready for our final

Theorem

For any maze M, using the normal strategy,

$$E(L) \leq S(M)$$

with equality if and only if M is of class A.

Proof:

Let a maze M have routes R_j, $j=1,\dots,p$. For each R_j we have a simple maze M_j where the paths in M_j are just those associated with R_j. If M is of class A, then, as above, $S(M_j)=S(M)$, but if not, then some components of $M-R_j$ may not appear in M_j and there is at least one j for which this is the case. So $S(M_j) < S(M)$ here.

Define the event A_j to be {a path associated with route R_j is chosen}, $j=1,\dots,p$. Using the normal strategy we have

$$E_M(L|A_j) = E_{M_j}(L) = S(M_j)$$

as M_j is simple, $j=1,\ldots,p$. But the A_j are disjoint and cover the whole sample space, so

$$\begin{aligned} E(L) &= \Sigma_1^p \text{prob}(A_j) E_M(L|A_j) \\ &= \Sigma\, \text{prob}(A_j) S(M_j) \\ &\leq S(M).\Sigma\, \text{prob}(A_j) = S(M) \end{aligned}$$

with equality iff M is of class A. QED

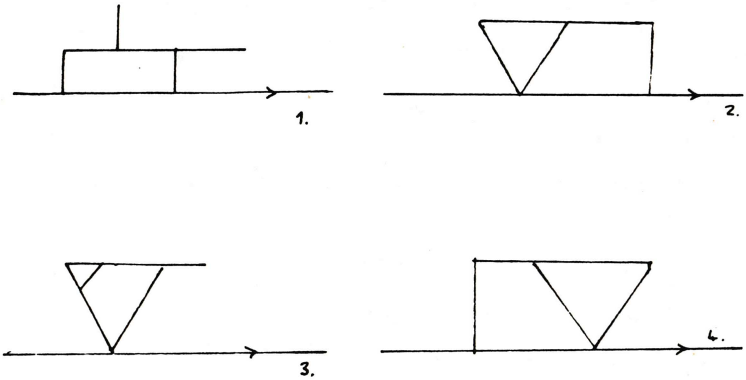


Figure 1.

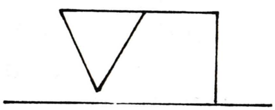


Figure 2.

PROBLEMS

by A. Hadden

The following problems were set in the Problems Drive in 1975. The answers will appear inside the back cover.

1. A mean tennis-ball manufacturer would like to know the smallest fraction of his warehouse space he has to waste in storing his stock. He does not spend money on boxes! (Ignore edge-effects)

2. In this sum each letter represents a decimal digit, not necessarily all dis tinct. Given that TWELVE is divisible by 5,7 and 12, decode the sum. (There are no leading zeroes)

```
        F I V E
      S E V E N +
    -------------
    T W E L V E
```

3. In Poker, '3 of a kind' is ranked higher than 'two pairs'. Justify by showing that it is less likely to occur. ('3 of a kind' includes e.g. 8S,8H,8D,2S,KS but not 8S,8H,8D,3C,3H)

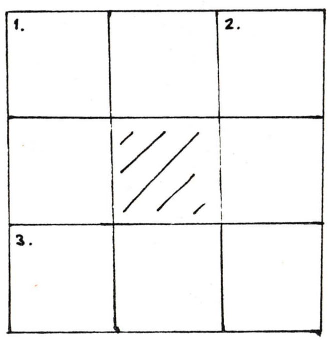
4. Solve this cross-number.

Across

1. The middle digit is half the mean of the other two
3. The final pair of digits forms three times the first

Down

1. First digit prime. Last two form its square
2. A multiple of 111



5. In how many ways can a golfer score 72 on an 18-hole course,

6. In a certain country the coins have values 1,2,4,8,16. In how many ways can a sum of 16 be made?

7. What is the next number in each of the following?

i) 6,2,8,2,10,? - it is not 2 !

ii) 10,11,12,13,14,20,22,101,?

iii) 3,4,6,8,12,?

8. How many squares are there whse edges coincide with the squares of a standard 8×8 chessboard?

9. Six houses lie on the vertices of a regular hexagon with an elect ricity plant at the centre. Indicate how to supply each house with power, using the shortest possible length of cable. You may introduce new nodes.

10. 2n cuts are made at random in a stick. What is the p probability that the (n+1)th section is at least half the length of the stick?

11. A jet fighter on the surface of the Earth is being c chased by two identical fighters, all with ample fuel. Can the pursuers get within shooting range, no matter how they start out?

12. In this version of the game MOO the object is to guess a number comprising three dis tinct digits in the range 1 to 7. The response to a guess is "m bulls and n cows" indicating m correct digits correctly placed, and n further correct digits incorrectly placed. Win this game -

Guess	123	Response 0 bull 1 cow
	456	0 bull 1 cow
	257	0 bull 1 cow

MECHANICAL GEOMETRY

by Clint O'Vou

This note describes a few methods of solving some problems of a very **pure nature** using appiled methods. The basis of these is 'extremalising' some quantity, which is done by 'pulling' the configuration suitably, and analysing the statics of the final situation.

A fairly well-known example concerns a triangle: we wish to find the point such that the sums of the distances from it to the vertices is minimal. This can be found by winding a piece of string around pins at the vertices and through a ring in the middle (all smooth - this is mechanics!) and pulling. (Fig. 1) The tension in all the parts of the string must be equal, so the required point is one where all three sides subtend an angle of $2\pi/3$, or at one of the vertices if this is impossible (e.g. at an obtuse angle if it exceeds $2\pi/3$). The point minimising the sums of squares of distances is well-known to be the centroid - put equal masses at the vertices and use the parallel axes theorem perpendicular to the plane of the triangle.

If we want to extremalise area, the obvious mechanism to use is surface tension, because the work done in changing the shape of, say, a soap film depends solely on the increase in area, and does so linearly. Accordingly, if we wish to find the maximum area that can be enclosed by a given set of line segments of of varying lengths, we can consider them arranged in any order in a loop, surrounded by a soap film; we imagine little hooks at the ends of the rods, so that we do not have to employ the full length of each. (Fig.2) First look at the forces on a single rod; these will be a force normal to it and proportional to its length, through its midpoint, due to surface tension, and reactions at its ends. These reactions must be equal in magnitude because they meet on the perpendicular bisector. Now consider the forces on the whole set of rods - the net forces are just the surface tensions, and if we draw their vectors rotated through a right angle, we will simply get the original diagram. We can then superimpose the triangles of force for each rod, and we find that the two (equal and opposite) reactions at each join between two rods are represented by the same line in this new diagram. So, since all these reactions are equal in magnitude, we see that there is a point inside the configuration, equidistant from all the vertices; therefore the vertices lie on a circle in the extremal configuration. The full length of each rod will be used except possibly for one if it would otherwise force all the vertices to lie on a minor arc; instead, just enough of its length to form a diameter is used. This is because if a rod is not crossed at its end, the reaction must be normal to it, and then when the force is rotated through a right angle it will be along the rod; but we have seen that it passes through the centre of the circle. Noting that the oreder of the rods around the loop does not affect the area if the vertices are on a circle, we can characterise the maximal systems as those that are cyclic using the full length of every segment except possibly one, so that not all the vertices lie on a minor arc.

We can combine bits of string and soap films in some cases; for instance, if we want to find the rectangle with largest area when we prescr ibe the sum of three sides, we can imagine four rods with trammels forcing them to cross at right angles, and a length of string running round three sides. (Fig.3) Then by simply considering the forces in directions perpendicular to the sides, we find that the tension in the string is a constant times the middle of the three sides, and that it is also the same constant multiple of the other sides together - reactions at the trammels produce only couples and so can be ignored. Thus we must take on side as half the fixed length, and the other two as one quarter each.

In these and other similar applications it is important to consider the system for imposing the constraints carefully in order to ensure that they will be the right ones, and to discover their mechanical implications. However, once this is done, the approach often provides very neat results.

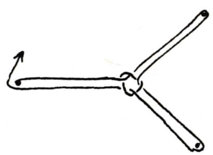


Fig. 1.

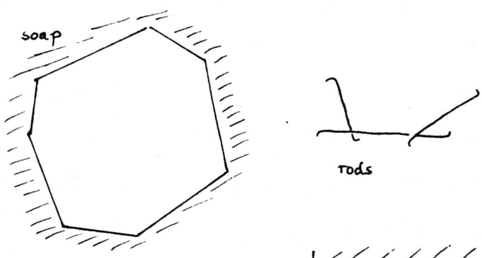


Fig. 2.

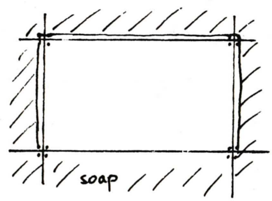


Fig. 3.

EXCHANGE

by Colin Vout

The puzzles and Games Ring of the Archimedeans is a subgroup which has benn in existence for more than ten years, with varying degrees of attendance. During the last few years, the 'puzzles-and-' part has been less in evidence and the playing and inventing of mathematical games has taken over. Exchange is one such game which has proved itself popular.

Exchange is a game for four players, each partnering the one sitting opposite him; it is played with a pack of 56 cards, comprising the four standard suits plus four jokers, two each of two distinguishable types (called 'red' and 'black' or 'left' and 'right'). The aim of the game is for one meber of a partnership to collect in his hand a 'straight flush' of six cards; ace counting high or low. For instance, any of S5678910, C10JQKA2, HA23456 would do. This aim is achieved by collecting cards from a central pool or taking them from other players (i.e. by fair exchange or robbery).

Eleven cards are dealt to each player and twelve to the central pool, called the "exchange". The exchange is dealt in a circle, with a gap bewteen the first and last cards; all cards are face-down and play commences on the dealers left.

On his turn, a player displays to all the others any card from his hand. If this is any but a king or joker, he then returns it to his hand and also picks up (without showing it to the others) the card from the exchange that corresponds in number to the one he has just displayed -

if it was an ace, he would pick up the first card, if a seven the seventh, a jack the eleventhand so on. He then puts in its place (face down) any card in his hand; this could be the card he first showed, the one he has just picked up or any other. If it was a king he first displayed then the card that was last in the exchange is moved into first place and the rest moved round one, so that the card corresponding to four at the beginning of the turn now answers to five. The king shown is then returned to the players hand. If, however, the card first displayed was a joker, he must then display two further cards, and address a request to one of his opponents for the third card that would form a three-card straight flush with the two just displayed; this request is directed at the player on his left if the joker was red, and on his right if black. So a player can ask for the C10 from his right opponent with B,c9J or for the H4 or H7 (but he must specify which) from his left opponent with R,H56. If the opponent has the card requested, he then exchanges it for the joker; otherwise no cards are transferred. If after his turn a player has won, he announces it - otherwise play passes to the left.

Throughout this game, players must be aware of what their partners are in need of; they must be able to exchange cards freely and attempt to stop their opponents from doing so. Because of this,it is useful for a pair to have a monopoly of cards of a given rank - e.g. if two partners have threes and their opponents have none, they can pass cards to each other freely by putting them in the third pile in exchange. If the opponents cannot obtain any threes, their only way to interrupt this is to use a king to move what was in the three-pile to the four-pile. To ascertain which numbers can be used for communication, it is customary for the first few rounds to be devoted by the players to showing cards of a rank they have two or three of and for partners to respond if possible with a card of the same rank. It is also useful to show a card in an area in which you are collecting, so that partner can pass across suitable cards if he has any; if this can be combined with showing the denominations this is especially helpful.

No further conventions have yet evolved, but it would clearly be a good thing to keep one's partner informed about ones own hand; however this might be at the expense of unproductive play, and the aim must be to strike the correct balance.

A sample game follows; the notation used is largely self-evident.

The deal:

R	B	B		
S 8	S 6	S 29Q	S 410J	
H 9	H 347	H 510J	H A268	4D R 5S 10D
C 46JQ	C 3510	C 28K	C 79	AS 3S KS 7S
D A6QK	D 27J	D 8	D 39	QH 5D AC KH

The play:

6D(3S-3S)	3H(5S-10C)	2C(R-5H)	3D(10C-2H)
QD(KH-8S)	3C(2H-6S)	QS(8S-8D)	3D(6S-AH)
QC(8D-QC)	3H(AH-JD)	8S(7S-JH)	10C(5D-5D)
AD(4D-9H)	4H(10D-10D)	BS89-	3D(JD-10S)
JC(AC-KH)	3C(10S-5C)	BS78-	3D(5C-6S)
KD	4H(6S-2D)	8S(KS-10H)	3D(5H-5H)
6C(AS-AS)	3H(5H-7D)	RSQK+J	3D(7D-6H)
KD	3H(9H-10S)	KS	4S(10S-8H)
4D(8H-6C)	5H(6H wins.		

It is theoretically possible for a side to be unable to win; for instance if their opponents hold A78 in all four suits and all four jokers, but it is to be presumed that in any such situation the other side is in a strong enough position to force a win. I have never known a game go on for more than twenty or so rounds, but it may prove necessary to legislate for this. If a pack with four jokers is unavailable, the game can be played with the kings also acting as jokers at will. It might also be fr fruitful to experiment with slightly modofied rules,such as an aim of collecting five, or even seven cards from exchange, or different rules for robbery. I would be very interested to hear about any such suggestions or the develoment of any major conventions.

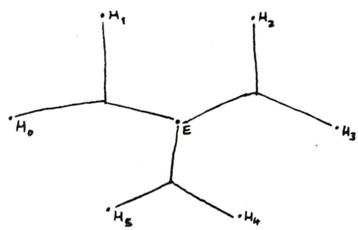


Figure for Solution 9.

SOLUTIONS

1. The fractional volume wasted is $1-\pi/3\sqrt{2}$

2.

```
      9 5 8 0
    9 0 8 0 0
  -----------
  1 0 0 3 8 0
```

3. Prob{3 of a kind} = $8\times11/5\times17\times49$

   prob{2 pairs} = $2\times9\times11/5\times17\times49$

4.

```
2 1 2
0   2
4 1 2
```

5. $\binom{71}{17}$.

6. 36.

7. i) 18 - digits of π doubled

   ii) 1010 - ten written to base 10,9,...,2

   iii) 14 - primes + 1

8. 204

9. (all angles 120^o)

10. $2\uparrow(-2n)$

11. Yes. The pusuers start at the poles and keep on the same meridian as the lone pilot, moving towards the equator

12. The possible answers are -

    374,671,714,734,741,761