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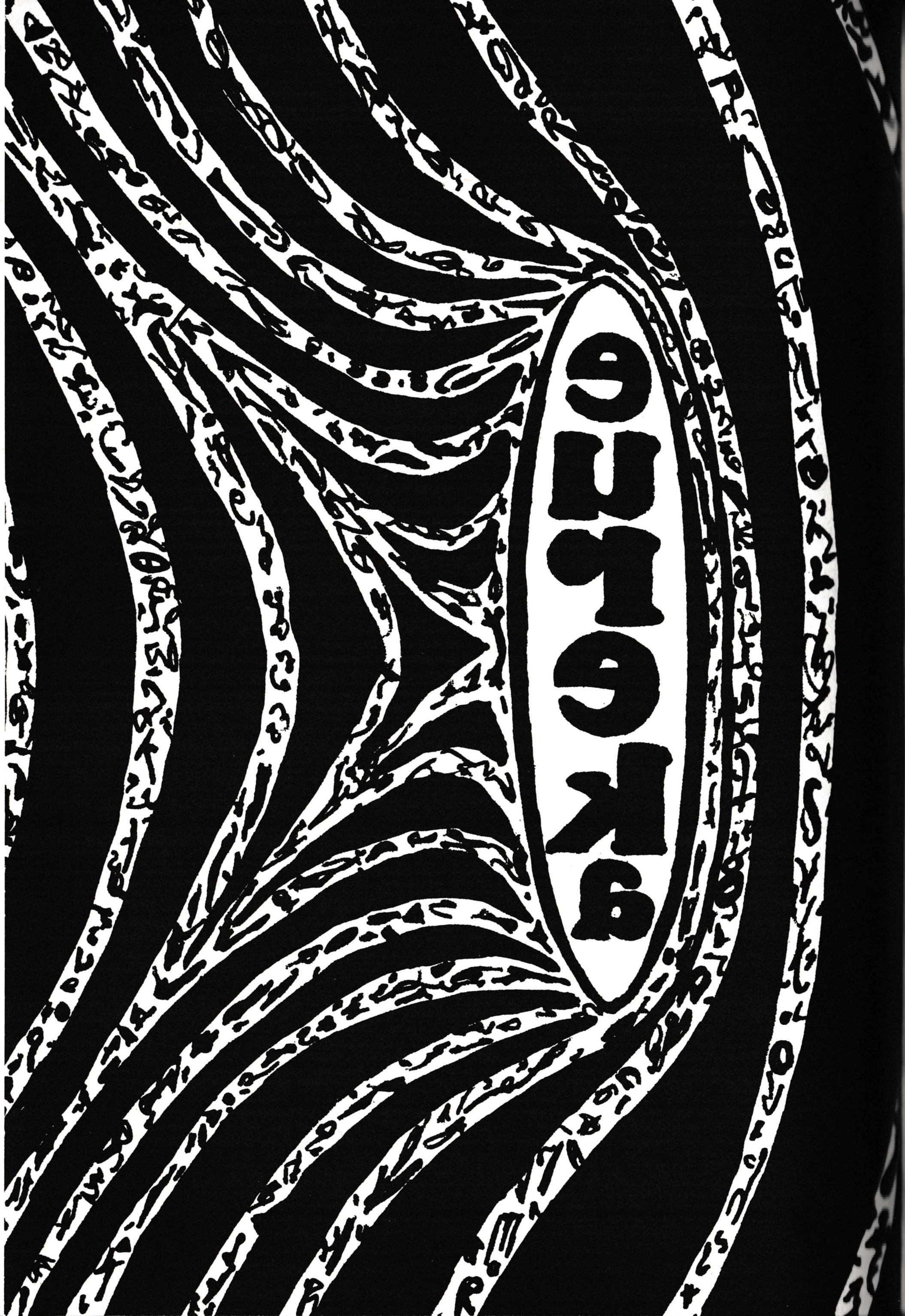

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EUREKA

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We must apologise for the step-function character of the price but owing to unbearable financial pressure, including the conspicuous lack of advertisers, an increase was unfortunately necessary, hopefully the last for at least four years. Postal rates: 22p unless £1.00 or more sent in advance in which case accounts will be debited at 20p per issue (corresponding dollar rates are 75c per copy or 70c if more than $4.00 sent in advance. The attention of overseas subscribers is drawn to the fact that it costs Eureka 60c for every cheque cashed drawn on a foreign bank.)
Back numbers on request: few before 1960– xeroxed copies at 65p if not available.

The editor would like to thank all those who have helped in the production of this issue, especially my predecessor Anthony Kemner, Rodney Brewis, Joseph Conlon, Guy Lucas and Mike Merrington for the cover.

EDITORIAL

Whilst thumbing through an old copy of Milne-Thomson's Hydro-dynamics I was interested to see that he uses the verb 'to burble' in its true and original meaning, that is, the turbulent behaviour of a fluid flowing past a sharp corner. It is good to see that a few are still using the old classical terminology which triste dictu, seems to have passed away prematurely. However, I was prompted by this to remember a time in my extreme youth when I was first initiated into the chiaroscuro world of mathematics, and when I first learnt the meanings of those words which, sadly, the majority of mathematicians today seem to have forgotten.

It was a sultry summer's afternoon, just before tea-time, and my aged parent and I drifted down the slowly flowing river at Egdirbmac in a curious craft called a t'nup (unfortunately not, as its name suggests, capable of travelling at 100 mph), discussing, naturally enough, the general motion of an axisymmetric body fixed at a single point (believe me, it was absolutely topping)

"You see," warbled my aged parent, "we assume that the tove (i.e. the axisymmetric body, as we now call it) is smooth, that is to say completely slithy, and is fixed in a gimbal which itself can rotate in another axis called the wabe."

"Why do you do that?" I crooned ecstatically, hardly able to contain my childish curiosity.

"Clearly, or brillig as we mathematicians say, to enable the tove to gyre freely." was the patient reply, obviously delighted at my apparent ignorance. "Now the fascinating thing is that the borogoves, (the joint plural of herpolhode and polhode) roll on each other, that is to say they mimse. If you like, the borogoves are mimsical or mimsy, as we sometimes say."

Aged parent changed his expression to one of seriousness and in an inspirational tone of voice said.

"If you remember all that, my lad, you'll produce some great mathematics one day, perhaps even as great as the Rome maths where all this was first grabed out." (I have since learnt that these facts were first discovered in that Italian city).

So inspired with this was I that I was determined to remember it all. But, owing to my over-enthusiam I became somewhat muddled and so as the memory of what my father had said was fast evacuating my mind, I composed a mnemonic in the form of a rhyme. As I was only six the spelling was not quite right, but here it is.

'Twas brillig, and the slithy toves
   Did gyre and gimble in the wabe;
All mimsy were the borogoves,
   And the mome raths outgrabe.

Strange, but I cannot help feeling that I have seen it somewhere before.

THE HISTORY OF AN INVENTION

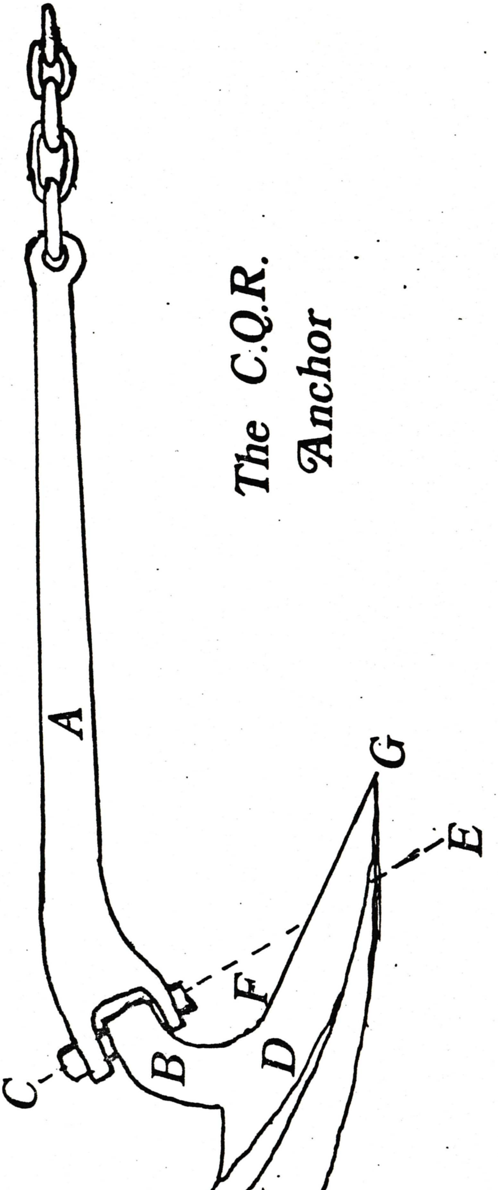
G. I. TAYLOR

Archimedes was not only a mathematician who expressed his thoughts by means of figures written on sand but, as your name 'Eureka' reminds us, solved some essentially mathematical problems without using figures or symbols. When your Editor asked me to make a contribution to 'Eureka' to commemorate the forthcoming publication of the last volume of my collected papers it seemed a good opportunity to describe the history of one of them which contains no symbols.

In 1923 I bought the 48 foot yacht 'Frolic' which weighed 20 tons and drew seven feet of water. Her big anchor weighed 120 lbs. While winding up the anchor the sails were not capable of controlling the boat until the anchor was nearly up to the surface and when anchored in 10 fathoms or more closer inshore with an onshore wind the effort involved in winding up the anchor to get under control before drifting ashore was too much for me. This and some problems connected with seaplanes provided the incentive to think about the design of lighter anchors.

The earliest anchors were simple stones so that the ratio holding power/weight, (H/W), was less than the coefficient of friction measured in air, usually less than one. The Greeks realised that a much bigger H/W could be attained by using a hook which would dig into the ground and they, or their contempories, invented the stock, that is the long bar at right angles to the plane of the hook which prevents it turning out of the ground once it is in. Since the stock would hold the fluke (that is the bent up part of the hook) pointing upwards if it fell that way and so prevent it from acting, it was necessary to add a second hook on the opposite side of the shank, thus making the anchor symmetrical about two planes through the shank. This second hook is necessarily at such an angle to the hook that it prevents the first from dragging the shank downwards. For this reason the high values of H/W which could perhaps be attainable by a single hook cannot be had with a traditional anchor.

My problem was therefore to think of a way in which a single hook without a stock could be made to dig into the seabed which ever way it fell, and be stable when pulled horizontally below the surface. The solution I came to is shown in the sketch. The shank A is hinged to the fluke B by a pin C whose axis is shown by the broken line CE. The blades D and J are nearly portions of circular cylinders with a common generator FG. The sketch shows the anchor seen from above and lying as it falls with A, J and G on the ground. When the chain pulls, the point G begins to dig in because it is aiming obliquely downwards, and the lateral pressure turns the blades further downwards because the centre of lateral pressure is ahead of the line CE. As the blade buries itself the centre of lateral pressure moves backwards and when it passes the line CE the direction of rotation of the blades about the pin C reverses and after dragging a short



The C.Q.R. Anchor

distance the anchor assumes a position where the plane of symmetry is vertical. In this position the blades can pull the shank into the ground. Also the anchor is stable when pulled with horizontal shank and blades under the ground, for if it rolled slightly so that the blade J was lower than the blade D, J would be in ground which was deeper and therefore more difficult to move than that round D. Thus the blades would rotate about the pin in such a way that the point G turned downwards and the anchor would return to the symmetrical position. By experimenting with a model on a sandy beach I found that the anchor could be towed in a circle keeping under the surface. When I dug up the blades while performing this experiment I found the blades banked over just like an aeroplane when it makes a turn, but remained symmetrical when pulled in a straight line.

The maximum value of H/W varied with the nature of the seabed, in some grounds H/W was over a 100 which is four or five times as great as that attainable with the traditional stocked anchor and 20 times that of the stockless anchors which all big steamships carry.

After inventing the anchor I, together with my friends George McKerrow and W. S. Farren, set up a small company to make them for our sailing friends. Farren undertook the making of drawings suitable for supplying to a manufacturer, McKerrow arranged the marketing and I gave the invention. We called the company 'The Security Patent Anchor Co.' and would have liked to put the word 'secure' on the anchor, but it is not allowable to register a common word in that way, so we compromised and called our product 'C.Q.R.' For a long time afterwards people asked me what the Q stood for.

The history of this small company is instructive. It was founded in 1933 or 4 and had only begun operations and declared a small dividend of 5% on the minute capital of £900 in 1939 when war broke out. Since the anchor was only intended for yachts we expected our operations to close down but soon the admiralty started ordering them for their torpedo boats and George McKerrow who had taken on the job of managing director was kept busy through the war. Finally these were copied by Lord Mountbatten's combined operations group and used to anchor the floating 'Mulberry' harbour from which the Normandy landings were launched in 1945.

When the war broke out the Government passed anti-profiteering legislation which made it illegal for companies to increase their dividends and limited the fees payable to directors for attending to the business of companies. This was hard on a company like ours which had only just reached a stage at which there were any profits at all and it had effects which cannot have been intended. For instance, the anchor began to be used for purposes for which it had not originally been designed and the company was asked for advice. In some cases I made experiments to supply an answer. In other words I acted as a consultant, but as I was a director I was not allowed to charge for my advice. Nor was I allowed to resign from being a director to become a consultant, because according to a legal adviser that would have been regarded as a shady transaction, even though I received in fact no payment as a director. Since the capital of the company was only £900 such fees would have been minute anyway.

After the war a grant was made to the company for the use of the patent during the war though we did not ask for it. The grant was small since I had given the invention to the company, so that it had no assignable value. Even so most of it would have been absorbed as income tax if it had been distributed. In any case the patent had only a few more years of life and all of us had other things to do, so we sold the company with the grant included in its finances to a firm which could use the grant for development. It still makes C.Q.R. anchors.

ANALYSE MATHEMATIQUE DES FORMES DES MONNAIES

PAR PAUL ET NAOMI LAGUERRE

La forme traditionelle des pièces de monnaie est un cercle parfait, $x^2 + y^2 = r^2$; même les Romains avaient des monnaies rondes, à peu près circulaires. Au contraire, certaines pièces de monnaie britanniques contemporaines ont d'autres formes, dodécagonales et heptagonales. On se demande pourquoi les monnaies ne sont jamais carrées. Or, ça produirait un grand malheur. Car on connaît bien la phrase, "L'argent est racone de tous les maux", c'est-à-dire, l'équation

l'argent = $\sqrt{}$ (tous les maux) (1)

On en déduit donc tout de suite que

l'argent au carré = tous les maux (2)

une conséquence peu désirable.

QUICKIE

M, M + 1, M + 3, and M + 4 is a sequence of integers each the sum of two squares, of which $M = n^2 + (2n + 1)^2$ Deduce the remaining 2-squares and thus find M.

EXAMINATION TECHNIQUE

C. J. MYERSCOUGH

It is often said that examination results provide little guide to academic ability; that they are much affected by candidates' differing 'skill at passing examinations'. One might infer that some candidates have sacrificed their academic innocence in the study of a well-established black art, whose copious literature is the result of extensive research. Yet nothing could be further from the truth; no systematic study of examination technique has been made, nor even has there been any attempt to apply the general results of noted authorities in related fields. With so much multi-disciplinary research going on now, this neglect is surprising, for as we shall see the subject offers an interesting combination of statistics, optimisation theory, and psychology. These notes merely indicate a few possible directions of research.

The subject divides at once into *strategy* and *tactics.* The former is about the planning of one's academic work throughout the year in order to maximise one's expected examination performance, whilst the latter concerns the examination period itself.

The most important strategic problem is that of the multi-course (or multi-subject) examination. M courses are given during the year, and a particular candidate spending time t_m on the m'th course throughout the year can expect to obtain $x_m(t_m)$ marks in the examination. He does not wish to spend more than time t in all on his academic work. How should he arrange his studies in order to maximise his aggregate expected performance? That is to maximise

$$x_1(t_1) + x_2(t_2) + \ldots\ldots + x_M(t_M) \quad (1)$$

subject to the constraints

$$t_1 \geqslant 0\,,\, t_2 \geqslant 0\,, \ldots\ldots, t_M \geqslant 0 \qquad (2)$$

$$t_1 + t_2 + \ldots\ldots + t_M \leqslant t \qquad (3)$$

The ratio $\frac{1-t}{t}$ is known as the *indolence factor.*

Clearly the inequality in (3) may be replaced by an equality. Further constraints of two types might be required; some to express the dependence of the work of one course on that of another; others to take account of the examination of different courses on the same paper, with a consequent combined time limit. In many cases, for example in the Mathematical Tripos Part II, the first type of constraint may be ignored; later we shall advance strong arguments for neglecting the second type also.

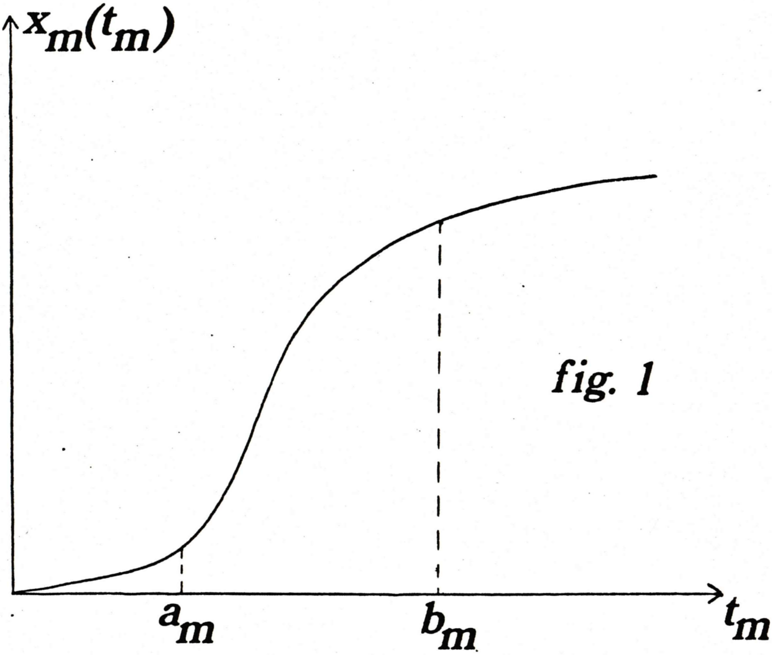


fig. 1

In some arts subjects, where the examinations test almost entirely the assimilation of reading lists, useful results might be obtained by taking the x_m as linear functions, but this is certainly not true for mathematics. The form of $x_m(t_m)$ is instead roughly as shown in figure 1. A basic comprehension of most of the course-work is normally required to answer any of the questions, and this takes time a_m to achieve; x_m therefore remains small for $t_m \leqslant a_m$. Further practise, up to a total time b_m is required to get the knack of doing Tripos questions on the subject quickly. But this additional work pays greater dividends; x_m increases more rapidly. Typically, $x_m(a_m)$ might be 10 – 20%, and $x_m(b_m)$ around 70%, of the total marks available for the course. For $t_m > b_m$, x_m increases more and more slowly; each step towards perfection becomes more difficult. The a_m and b_m are of course a measure of the candidate's ability at each subject.

The equations (1) – (3) specify a fearsome-looking non-linear programming problem, but its approximate solution is clear if we consider what happens as t increases from zero. Let the courses be numbered in order of the candidates proficiency, so that $b_1 \leqslant b_2 \leqslant \ldots\ldots \leqslant b_m$ Then for:–

$0 \leqslant t \leqslant b_1$; $t_1 = t$., all other t_m zero.

$b_1 \leqslant t \leqslant b_2$; $t_1 = b_1$, $t_2 = t - b_1$, all other t_m zero.

$b_2 \leqslant t \leqslant b_3$; $t_1 = b_1$, $t_2 = b_2$, $t_3 = t - b_1 - b_2$, all other t_m zero.

And so on up to $t = b_1 + b_2 + \ldots\ldots + b_M$. For larger t, the solution is more complicated, but few candidates will have the ability or diligence to put them in this range; furthermore the limited time available in this examination will almost certainly now be an important constraint.

One can therefore sum up the best strategy for the Mathematical Tripos as follows: a candidate should take as many courses as he can prepare up to a fairly good standard, rather than fewer up to a better standard, or more up to a worse standard.

In fact, the candidate does not usually have a very good idea in advance of what courses will be most suitable for him – though supervisors and directors of studies should be able to help. The best dynamic strategy will involve taking more courses to begin with and abandoning some during the year – though much research is required to suggest exactly how this should be done.

Turning now to the tactical problems of the examination itself, we note that examinations fall into two types. If

$$N \gg C^2 \qquad (4)$$

where N is the number of candidates taking the examination, and C is the number of approximately equal classes into which the candidates are divided, considerations of the variance show that fluctuations in the performance of one candidate do not affect the assessment of another. Thus each candidate may ignore the others and regard himself as being assessed according to a fixed average standard. The examination is *perfectly competitive.* Most Cambridge examinations fall into this category, though the competition for research places in particular subjects usually does not. The range of tactics available is far smaller than for imperfectly competitive examinations, to which may be applied the comprehensive theory of games developed over the past 40 years by Potter and others.

Indeed, the only useful tactic in a perfectly competitive examination is to use all the time available. The fact that many candidates neglect the most obvious means of doing this is what leads us to suspect that examination time constraints are unimportant in our discussion of strategy. At least five, and possibly ten minutes of extra time may be gained in a three hour examination as follows:-

1) Arrive at the examination room 15 minutes before the start. Not only does this allow some margin for delays on the journey; it also ensures that one will be among the first to enter the room. Normally one can start as soon as one sits down.

2) Bring at least two filled pens into the examination.

3) Do not fill in headings during the examination, apart from question and sheet numbers to avoid confusion. One can stay as long as one likes at the end to do this and to sort sheets into bundles etc.

In conclusion, tactics can be summarised in the statement 'First in, last out!'

ON $\frac{22}{7}$ AND $\frac{355}{113}$

D. P. DALZELL

For two thousand years the approximation to the value of π,

$$3\tfrac{10}{71} < \pi < 3\tfrac{1}{7}$$

has been the most famous approximation in all mathematics, but until 1944 no direct demonstration of it was available for text-books. Such a demonstration must be rigorously elementary and satisfactorily brief. Both these conditions are satisfied by the following process.

We have exactly, $\pi = \displaystyle\int_0^1 \frac{4dt}{1+t^2}$

and the denominator 7 suggests integration of a polynomial of degree six. If we add to the numerator, 4, a polynomial of degree 8 that is always small in (0,1) and makes the result divisible by $1 + t^2$ these conditions will be satisfied. Such a polynomial would be $A\{t(1-t)\}^4$. We find by division,

$$\{t(1-t)\}^4 = (1+t^2)(t^6 - 4t^5 + 5t^4 - 4t^2 + 4) - 4 \qquad (1)$$

Alternatively, $4 + A\{t(1-t)\}^4$ would have to vanish if $t = \pm i$. This gives the same result. Then,

$$\pi = 4\int_0^1 \frac{dt}{1+t^2} = 4 - \frac{4}{3} + 1 - \frac{2}{3} + \frac{1}{7} - \int_0^1 \frac{t^4(1-t)^4 dt}{1+t^2}$$

and thus,

$$\pi = \frac{22}{7} - \int_0^1 \frac{t^4(1-t)^4 dt}{1+t^2}$$

The value of the integral is between the two values obtained by substituting 1 and 2 for the denominator of the integrand, so that,

$$\frac{1}{1260} < \int_0^1 \frac{t^4(1-t)^4\,dt}{1+t^2} < \frac{1}{630}$$

and consequently,

$$\frac{22}{7} - \frac{1}{630} < \pi < \frac{22}{7} - \frac{1}{1260}$$

This implies the approximation obtained by Archimedes.

Since the time of Newton the representation of numbers by infinite series has become commonplace in mathematics. To avail ourselves of this novelty write

$$P(t) = 4 - 4t^2 + 5t^4 - 4t^5 + t^6$$

$$X(t) = t(1-t).$$

Then the identity (1) can be rewritten in the form:

$$\frac{4}{1+t^2} = \frac{P(t)}{1+\frac{1}{4}(X(t))^4}$$

whence by integration,

$$\pi = \int_0^1 \frac{P(t)}{1+\frac{1}{4}(X(t))^4}$$

Here the denominator of the integrand is symmetrical about ½ and the part, $Q(t)$, of $P(t)$ which is symmetrical about a ½ is found, by an appropriate calculation, to be,

$$Q(t)=\tfrac{1}{2}P(t)+\tfrac{1}{2}P(1-t)=3+X(t)-\tfrac{1}{2}X(t)^2-X(t)^3 \qquad (2)$$

and accordingly,

$$\pi = \int_0^1 \frac{Q(t)dt}{1+\frac{1}{4}(X(t))^4}$$

The expansion of the integrand as an infinite series integrable term-by-term is trivial and,

$$\pi = \sum_0^\infty a_n = \sum_0^\infty (-\tfrac{1}{4})^n \int_0^1 Q(t)(X(t))^{4n}dt \qquad (3)$$

where,

$$(-4)^n a_n = 3\frac{((4n)!)^2}{(8n+1)!} + \frac{((4n+1)!)^2}{(8n+3)!} - \frac{1}{2}\frac{((4n+2)!)^2}{(8n+5)!} - \frac{((4n+3)!)^2}{(8n+7)!} \qquad (4)$$

The terms in this series are less in magnitude than those of a geometric series with a common ratio $\frac{1}{1024}$ but the terms themselves are regretably complicated.

We note that by (2)

$$3 \leqslant Q(t) < \frac{13}{4} \quad , \text{ when } 0 \leqslant t \leqslant 1 \tag{5}$$

and that therefore from (3).

$$\frac{3\,((4n)!)^2}{4^n\,(8n+1)!} < |a_n| < \frac{13}{4^{n+1}}\,\frac{((4n)!)^2}{(8n+1)!} \tag{6}$$

Also, since Q(t) is positive in the interval $0 \leqslant t \leqslant 1$, we have,

$$|a_{n+1}| < \frac{|a_n|}{1024} \quad , \quad n \geqslant 0 \tag{7}$$

After $\frac{22}{7}$ the next well known approximation to π is $\frac{355}{113}$ due to Tsu-Chung-Chich more than six centuries after Archimedes.

The denominator 113 is prime, the smallest factorial divisible by 113 is 113! and the earliest term in the series (4) which has a denominator divisible by 113 is, at best, a_{14}, which for our purpose is much too late.

We have however, by calculation of a_1

$$a_0 + a_1 = \frac{22}{7} - \frac{19}{15015} = \frac{47171}{15015}$$

and,

$$\frac{355}{113} - \frac{47171}{15015} = \frac{2}{1696695} \tag{8}$$

But this comparison assumes that the new approximation is already known. If this were not the case resort could be had to the expression of $a_0 + a_1$ as a continued fraction, and we have,

$$\frac{47171}{15015} = 3 + \frac{1}{7+}\;\frac{1}{15+}\;\frac{1}{1+}\;\frac{1}{65+}\;\frac{1}{2}$$

with the convergents

$$3, \frac{22}{7}, \frac{333}{106}, \frac{355}{113}, \frac{22408}{7451}, \frac{47171}{15015}.$$

Considered as approximations to $a_0 + a_1$ the most economical convergents, relative to accuracy, are those corresponding to quotients immediately preceeding an appreciably larger quotient, that is, the convergents $\frac{22}{7}$ and $\frac{355}{113}$, corresponding to the quotients 7 and 1. For the remainder of the series (4) after the terms a_0 and a_1 we have by (7).

$$\frac{1022}{1023}a_2 < \pi - (a_0 + a_1) < a_2$$

and by use of (8),

$$\frac{355}{113} + \frac{1022}{1023}a_2 - \frac{2}{1696695} < \pi < \frac{355}{113} + a_2 - \frac{2}{1696695}$$

also, by (6)

$$\frac{3}{16}\frac{(8!)^2}{17!} < a_2 < \frac{13}{64}\frac{(8!)^2}{17!}$$

The inequalities,

$$\frac{13}{64}\frac{(8!)^2}{17!} - \frac{2}{1696695} < -\frac{24}{10^8}$$

$$\frac{1022}{1023}\frac{3}{16}\frac{(8!)^2}{17!} - \frac{2}{1696695} > -\frac{33}{10^8}$$

enable us finally to assert that,

$$\frac{355}{113} - \frac{33}{10^8} < \pi < \frac{355}{113} - \frac{24}{10^8}$$

BEAT THE CLOCK!

Rearrange the following into a well known phrase or saying:-

a) $\exists\delta\ \forall$ s.t. $f(x) - o < o - \epsilon \Rightarrow x_o > f(x_o) < x > \epsilon \|\delta\|$

b) $\forall [a, b]$ s.t. $N > N, \exists$ m, $\forall x \in fn(x), \epsilon\text{-}\epsilon > m > n$
no, $\forall$ x(m) $\|$ N<f

The results are both meaningful

SNAEDEMIHCRA ?

A certain (fictitious) Cambridge society has as its officers a President, a Secretary, a Treasurer and a committee of five – Anscombe, Edkins, Kelly, Rosenstiel and Williamson. Despite any exterior objectives the society may have, the objective of all members of the power structure is to become president as soon as possible. Constitutional niceties mean that any candidate for President, Secretary or Treasurer must previously have been on the committee. It is well known that in any contested election, the candidate with the larger power bloc backing him will win. If either Secretary or Treasurer opposes for President a candidate coming straight from the committee, their power bloc increases by 5 votes. The Secretary can count on the support of Anscombe and 3 other votes: the Treasurer on that on Edkins, Kelly and one other. It is known that the President has four votes behind him. Anscombe refuses to be allied with Williamson, and Williamson will not plot in any alliance containing Anscombe. There is a 50-50 chance that the Secretary and Treasurer will ally, with the Secretary running for President and the Treasurer for Secretary: otherwise they will stand against each other for President, with the President supporting either of them with probability ½. The President, whose initials are G.S.M., has red hair. Given a vacancy for either Secretary or Treasurer, the candidate of each alliance will be chosen by lot from amongst the committee members at that time members of the alliance. The President will not in general support alliances containing Williamson: and both Secretary and Treasurer will act in their own best interests knowing this. If the Secretary and Treasurer ally, the President may support them ($p = ½$) or set up a counter-alliance ($p = ½$): if he sets up a counter-alliance he will stand for re-election with probability ½, this being the only circumstance in which he will do this: otherwise he will run any allied committee member (even Williamson) with $p = ½$.

Advise Rosenstiel, who has two votes in his power bloc, and due to time requirements must make his alliance immediately – with President, Secretary, Treasurer, nobody etc., etc., – how to give himself the best chance of an officership.

The first correct solution of this problem to arrive at the Eureka office will be rewarded with a guided tour of the Eureka headquarters.

MAGIC MONEY

Form a 3 x 3 magic square with coins in current circulation so that each horizontal and vertical line, and the two main diagonals, add to 4p. There must be a different amount of money in each square and no square may contain no money.

A CAMBRIDGE LOOK AT LIFE

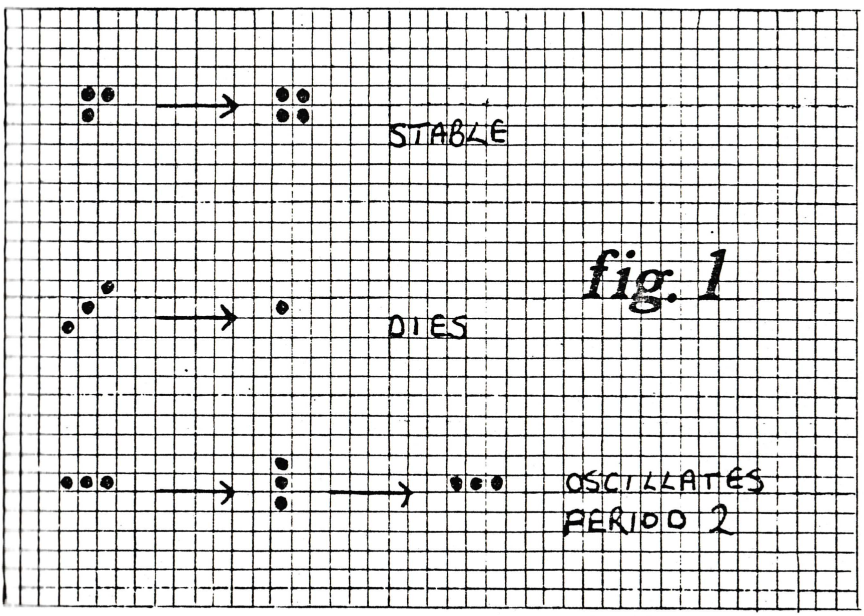
M. NEAVE AND C. HONES

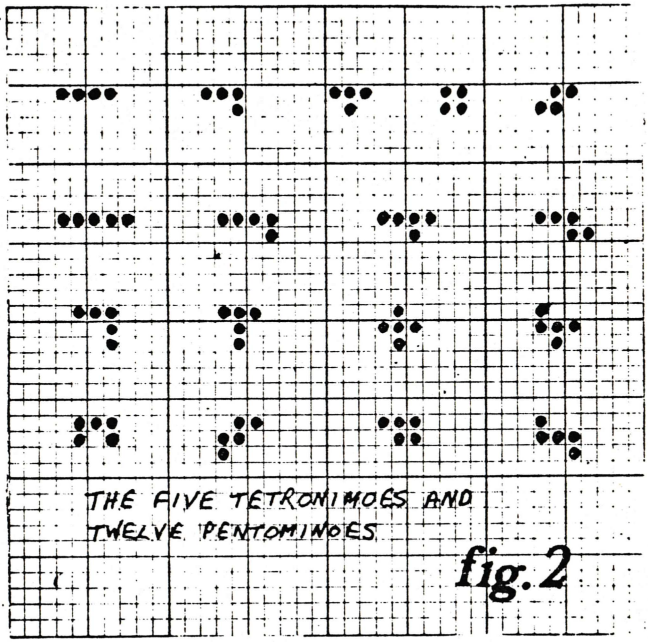
In the beginning Conway said, "Let there be Life". (sic)

What is life? Life is the rise, fall and alteration of a society of living organisms, as no doubt many readers will have already discovered. To justify this mathematically (and clearly it does need justifying) there is a growing class of simulation games which represent the real life processes formalistically. Of these Dr. J. H. Conway's game 'Life' is a particularly good example, so addictive, in fact, that never before in the field of mathematical endeavour has so much time been wasted by so many, investigating the consequences of so simple a set of rules!

Conway's Life is played on an infinite square grid of cells, each of which may be occupied or vacant, each cell having the 8 adjacent cells as neighbours. The game proceeds by generations according to the following genetic rules:-

1. Deaths: An occupied cell will survive to the next generation if exactly 2 or 3 of its neighbouring cells are occupied; otherwise it will die (vanish) i.e. cells die from either overcrowding or isolation.
2. Births: An empty cell will become occupied (i.e. give birth) in the next generation if it has exactly 3 neighbours (i.e. 3 occupied neighbouring cells)





These rules were chosen carefully after much experimentation in an attempt to make the game as unpredictable as possible; in particular, (a) there should be no initial pattern for which there is a simple proof that the population can increase without limit; (b) there should be initial patterns that apparently do grow without limit; and (c) there should be simple initial patterns that grow and change for a considerable period of time, possibly 100's of generations, before coming to an end. This can happen in 3 possible ways:-

(i) Dying completely, from overcrowding or isolation.
(ii) Stabilising, by reaching an unchanging pattern.
(iii) Oscillating.

These are demonstrated by a few simple examples in figure 1.

We suggest that in practice the following procedure for actually playing the game is the most efficient. Use a squared board (a go board is ideal) and a largish quantity of black and white counters. Start with an initial pattern of black counters and first locate where the births will occur in the next generation; place white counters in these squares. Next, locate counters that will die and cover them with another black counter, check thoroughly as mistakes are easily made even by experts. Finally, remove all double counters and replace white counters with black ones; you now have the next generation. One can also use a pencil and squared paper, this has the advantage of giving a complete life history of a particular organism, or, if you have one, a computer is ideal (and slightly faster).

For practice readers might like to trace the life histories of the 5 tetronimoes, 4 of which become stable and the fifth oscillating, and those of the 12 pentominoes, 5 of which die, 3 stabilise and 4 oscillate. (figure 2)

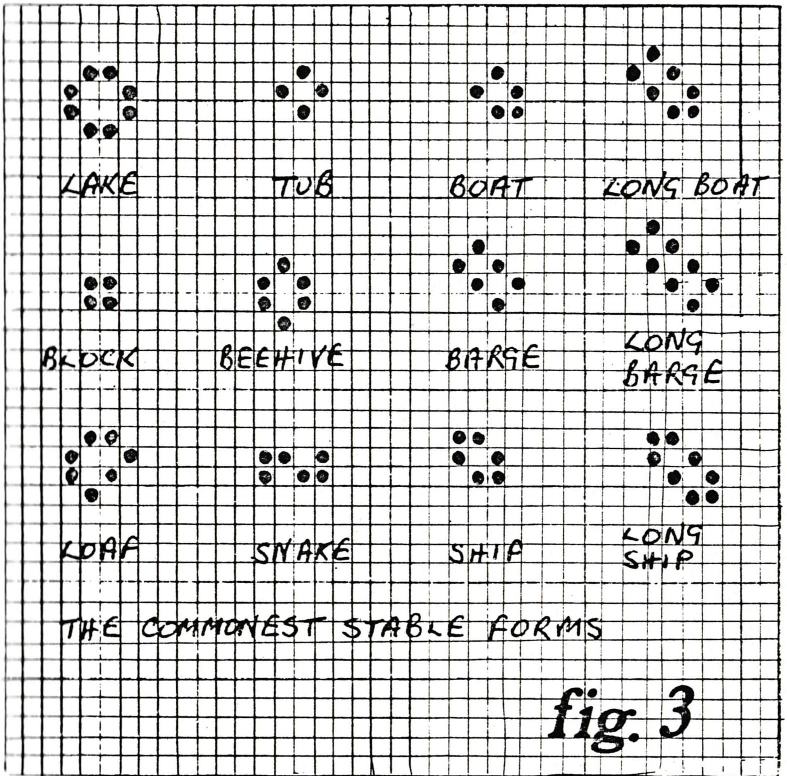
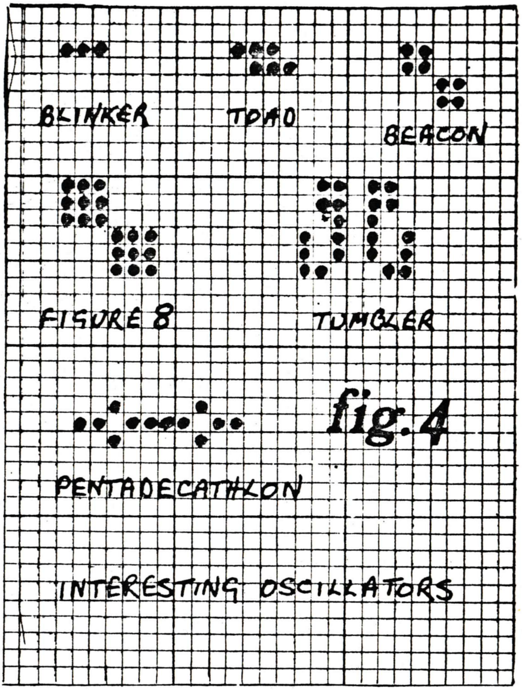


fig. 3

When investigating simple patterns it will be found that some final stable positions are fairly common. These have acquired their own names and are given in figure 3.

The simple oscillators, perhaps slightly more interesting, have also been labelled and are shown in figure 4.

Dr. Conway's initial conjecture was that there is no pattern whose population increases indefinitely, and a great deal of research has been done to find a pattern that does in fact do this. It was first realised that this could be done fairly easily when the 'glider' was discovered. This is a specialised type of oscillator with period 4 which travels diagonally across the board at a rate of one square every 4 generations. (figure 5). Clearly a pattern that fires off a glider will not die completely since the glider will always remain and there are many patterns which fire off large number of gliders before reaching a sad demise.



The question is, can we find a pattern that periodically fires off gliders and yet remains itself essentially unchanged? Well, yes we can, and this was first done by a group at M.I.T. who produced a glider 'gun' which fired off a glider every 30 generations. (fig.6). Hence, Conway's original conjecture was disproved since eventually the glider gun will produce an infinite number of bullets.

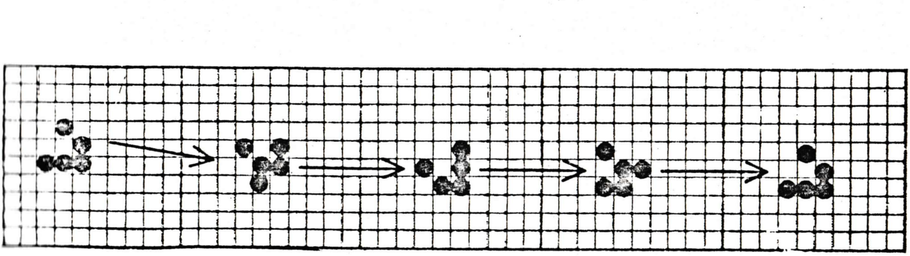


Figure 5. *The "glider"*

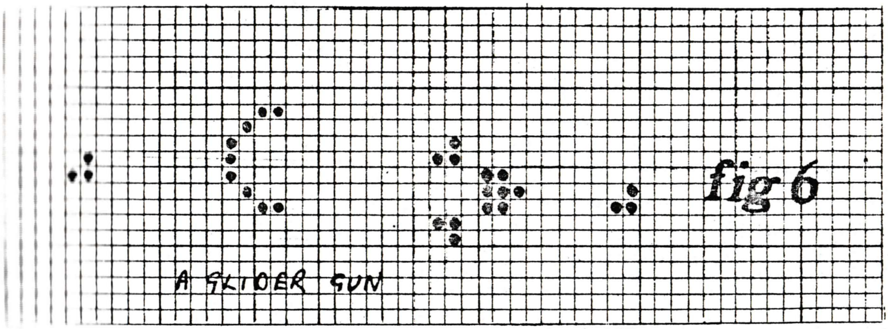
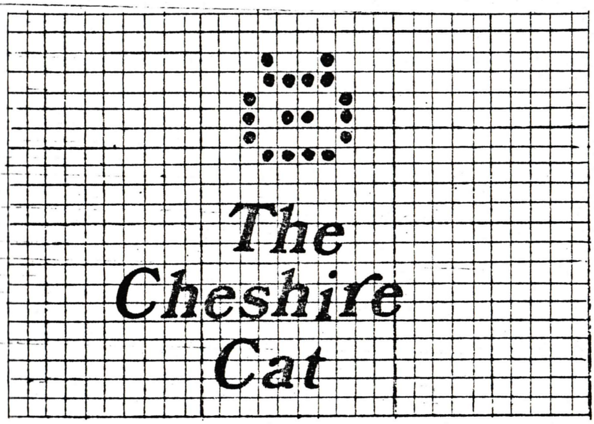


fig 6

We have a structure that will either reflect a glider through 180° or completely absorb it, and we are looking for one that refracts it. Another line of research is the hunt for a perfectly reflective surface, that is one that will reflect everything fired at it; so far not even a good reflector has been found nor even a good 'black body' i.e. one that absorbs nearly everything that hits it.

We have only space to mention a few of the hundreds of fascinating patterns generated by simple initial configurations (for instance, 'harvesters' trundle across the board leaving 'bales' behind them as they go – though a 'sower' is still to be found). Finally we leave you with the Cheshire cat which fades to a grin in six steps and then disappears completely except for a paw print.

Acknowledgements go to J. H. Conway and Martin Gardiner whose column in the Scientific American has given a much fuller account than is possible here. Enquiries are welcomed as well as any interesting discoveries.



SOME NOTES ON THE MATHEMATICAL WORK OF ALBRECHT DURER (1471 – 1528)

R. J. STEEMSON

May 21st this year marked the five hundredth anniversary of the birth in Nuremberg of Albrecht Durer. Although he must be chiefly remembered for his artistic work, including more than seventy paintings and hundreds of woodcuts and engravings, his contributions to science and mathematics must not be forgotten. These lie in his development of the applications of geometry and anatomy to art, and in his introduction of these ideas from Italy into Germany.

He began an apprenticeship to his father who was a goldsmith. However, by the age of 15 he had decided on a career as a painter and became apprenticed to the artist printer Michael Wolgemut. From 1490 onwards he travelled widely in Europe, although his home remained in Nuremberg where he married Agnes Frey in 1494, and died in 1528.

During this period the Renaissance painters in Italy had formulated the empirical laws of perspective and were beginning to apply the study of anatomy to their drawings. Durer travelled to Bologna to learn these laws of perspective and also spent two years studying in Venice. Not only did he introduce these ideas to his native Germany, but also he used his knowledge of geometry to treat perspective from a mathematical point of view. His theory of perspective was published in his book "Unterweissung der Messung mit der Zirkel und Richtscheid" in 1525. This book also contained chapters on linear geometry and on geometry in 2 and 3 dimensions. It contained results on conic sections and regular polygons and described the epicycloid for the first time.

In 1527 he published a book on theory of fortifications, and in 1528 an important work in which he laid the foundations of the science of anthropometry. Although his achievements tend to be eclipsed by those of his Italian contemporary Leonardo da Vinci, they are still of considerable importance, especially in their contribution to the spread of knowledge into central Europe.

QUEENS

Show how 5 Queens can occupy or attack the maximum number of squares possible on an 11 x 11 board.

Note: Queens attack squares in direct horizontal, vertical, or diagonal line to the square they occupy.

Further note: this further note may be ignored.

THE KNITTING OF SURFACES

BY M. O. REID

Introduction As anyone who has worn a woolly will know, knitting is rather a nice way of representing some 2-dimensional manifolds. The question naturally presents itself: which of the two-manifolds can be knitted without seams? Unfortunately the answer to this is rather dull: they all can be, although not all of them very nicely. Since knitting has got an obvious 'grain', knitting a two manifold provides us with a combing of it. So the question that is interesting is the following: which of the 2-manifolds can be knitted without seams and with only respectable singularities?

For example, let us look at the object below, a cylinder dividing in two. If we were trying to knit this, the branch point would have to have a singularity of the type shown.

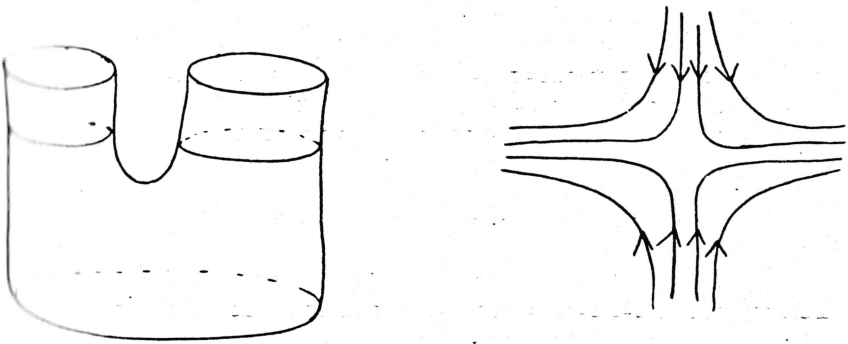


fig 1

This is something that cannot be very elegantly knitted: we will get a hole in the middle.

By contrast a singularity that does come off all right is the one that has been standard knitting practice since the invention of the bobble-cap. Starting with a cylinder we just decrease till there are only half-a-dozen stitches left, slip them all onto the thread, and pull tight.

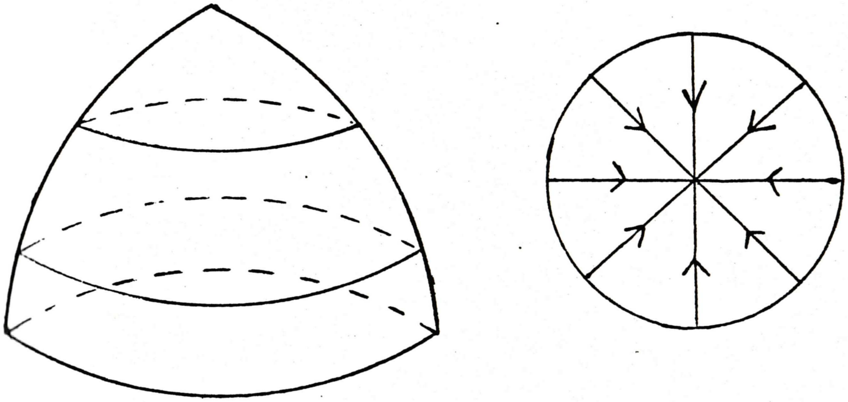


fig 2

This article provides patterns for the sphere, the torus, the klein bottle and one surface-with-boundary, the mobius strip. The torus and the klein bottle can be done with no singularity at all. The other surface than can be nicely combed is the real projective plane. I have a method for doing this but it is long and impossibly messy to describe. All the two-manifolds can now be knitted, by just taking connected sums of toruses and projective planes, using crude techniques, and of course the above untidy singularity.

Remark: the reader may wonder why I am always knitting on the round, rather than back and forth on rows. This is easy to explain: at any stage in the process, the piece of knitting is a 2-manifold with boundary. As is well known, the boundary now has to be a 1-manifold without boundary, i.e. a circle or collection of circles. This explains why I always start "cast on a cylinder of so-many stitches".

Technical Digression I require the use of three rather special techniques. Two of them are deduced from the appearance of a knitted cylinder. It is symmetric for reflections in a horizontal plane, and there is nothing to distinguish one row from any other. So one could have cast on a middle row first, and worked out; or alternatively, put the middle row in last of all.

The first is easy. Using spare (and different colour) wool, cast on a cylinder. Knit a couple of rows. Join in a main colour, and knit, say 10 rows. If the spare wool is now cut away, there is left a further row which can be slipped onto some further needles, and kept for future use.

The second is tricky, although a standard knitting technique (see the P. & B. booklet "Woolcraft" – the section on socks).

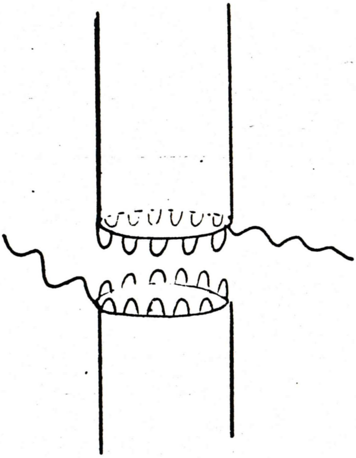
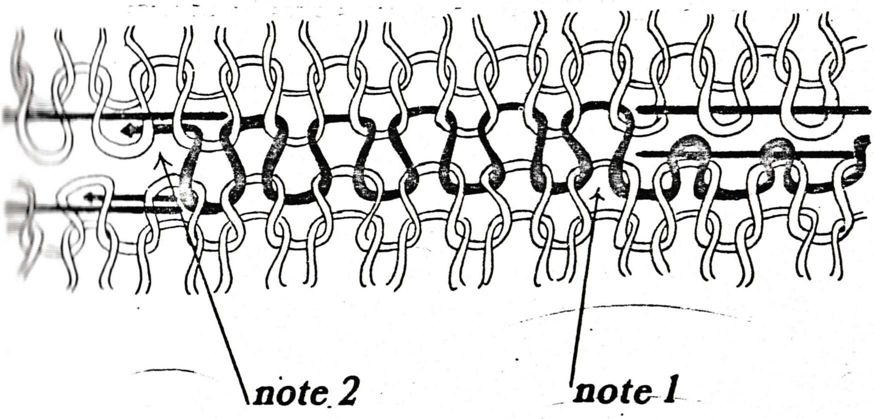


fig 3

As illustrated below, we have two cylinders with right side of work facing. The threads are at opposite sides of the cylinders, one of them being cut to a couple of yards, and threaded onto a bodkin. The process defies explanation, but I hope that the local diagram will make things clear.



In the local diagram note:

(1) The ~~general stitch~~ consists of one threading from front to back to front through the next stitch.

(2) The first stitch is perverse and confusing. The thread is passed through the next stitch from p. side to k. side of work.

I shall refer to this process as "grafting", since this is the standard terminology.

I do feel guilty that this is a cheat since it is "sewing up". I use the following remarks to satisfy my own conscience: a) it is standard b) it is dual to the two-sided casting on, which is irreproachable c) the purist who objects may say that I have cheated. but will not be able to say where, since the "grafting" row is in principle indistinguishable from any other row in which wool was joined, and is in practice indistinguishable, if the grafting was done carefully.

The third special technique is indispensable for making any of the non-orientable surfaces. Since they intersect themselves, we need a process for passing a cylinder through an already existing surface, a "wall". This is not very difficult, but requires a crochet hook. Slip the stitches onto a piece of spare wool, pull first the thread through a chosen hole, then each of the stitches, mounting them onto another piece of spare wool when they are through.

The Patterns requirements: set of four no.8 needles, two ounces of double knitting wool, a crochet hook, a few yards of a different colour scrap wool, kapok for stuffing (from Woolworth's).
The Sphere Cast on (both sides) a cylinder of 30 sts.

*k. 5 rounds, Decrease as follows.

Next round: (k.8 k.2 tog) 3 times
Next round: (k.7 k.2 tog) 3 times
Next round: (k.6 k.2 tog) 3 times
Next round: (k.5 k.2 tog) 3 times
Next round: (k.1 k.2 tog) 6 times
Next round: (k.2 tog) 6 times

Break off thread. Slip last sts onto thread and pull tight, leaving thread on wrong side of work.**

Join in thread at second side of casting on. Knit second hemisphere from *to**, stuffing firmly a few rounds before end if required.

The Torus (a) Cast on (both sides) a cylinder of 30 sts. Knit 80 rows, then half a round. Pick up sts from second side of casting on. Graft to finish, stuffing if required.

(b) Cast on (both sides) a cylinder of 32 sts. Knit one round.

Increase 8 sts in each of the next 8 rows as follows:
Inc in next st, k.3, inc in next st, k.3(5, .. 17) 4 times, knit 8 rounds, then *decrease* 8 sts in each of next 8 rows, as follows:
k.2 tog, k.3, k.2 tog, k.17(15 ... 3) 4 times, knit 7 rounds, knit half a round, graft off.

The Klein Bottle Cast on (both sides) a cylinder of 30 sts, knit 10 rounds. Increase 6 sts in every 4th row, 5 times, as follows:
1st (5th . . . 17th) row: k.4(5 . . . 8), inc in next sts six times (36, 42 . . 60 sts) others: knit. Knit three rounds.
Form a 'purl' window as follows:

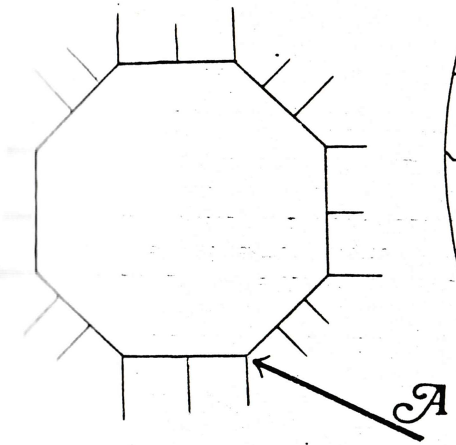
1st round: k.7, p.4, k. to end
2nd round: k.6, p.6, k. to end
3rd round: k.5, p.8, k. to end
4th–7th round: k.4, p.10, k. to end
8th round: as 3rd round
9th round: as 2nd round
10th round: as 1st round

K. 5 rounds, then first five sts of next round onto the end of the last needle, so that the round starts five sts later than previously.

Decrease 5sts every 4th round, 6 times, as follows:

1st (5th . . 21st) row: k.10, (k.8(7, ... 3) k.2 tog) 5 times (55,50 . . 30st) others: knit. K. 5 rounds, then first three stitches of next round. Pass thread through wall at "A" (in diagram) and pass cylinder through the wall at the outside edge of the purl window. K. 50 rounds. Graft off, stuffing.

The Mobius Strip This pattern requires a special weapon – a circular needle 32" long. The method is merely an improvement of "two-sided" casting on such that both sides of the stitches can be used from the beginning.
Using spare wool and a pair of needles, cast on 90 sts.



Using main wool, knit these onto the circular needle. They now cover about 2/3 of the needle, the stitches at the far end from the thread lying on the plastic. The working end of the needle is bent round to the far end to pick these stitches up, and these are *not* slipped off the end as they are knitted. When the row is all picked up, the needle loops the work twice, and the effect is as of ordinary two-sided casting on except that the second row is held on the needles. Knit into this row again. Knit five rows, and cut away the spare row. Cast off. Obviously it is not satisfactory to do a Mobius strip in stocking stitch! However, two-sided casting on cannot possibly work for any rib! (try it and see)

THE ARCHIMEDEANS

The Archimedeans have had a good year, with the evening and tea meetings well attended. The evening meetings included talks by Professor E. C. Zeeman on "Catastrophe Machines" and Professor H. Laster, on "The Propagation of Cosmic Rays". There was also a meeting in the Easter term at which Professor Marshall Hall spoke on "Problems in Arrangements". Tea meetings were very successful, with Dr. J. H. Conway on "Hackenbush, Welter, Prune and other games" and Dr. A. F. W. Edwards' talk: "Probability Theory and Human Genetics".

There was the usual visit to Oxford to play games with the "Invariants" and the Problems Drive in which the "Invariants" visited Cambridge. The visit to the Rutherford High Energy Laboratory at Abington was very successful. In the Lent term a dinner was held in the Graduate Centre. Amongst the society's guests were Professor Sir Nevil Mott and Professor J. F. Adams. The Computer Group has had an active year and the Music group and the Bridge group have both met frequently, but the Puzzles and Games Ring died in the Lent Term and its future is uncertain. The Bookshop has continued to thrive.

Speakers for the coming year include Professor C. T. C. Wall on "How to Organise a Tournament". Lady Jeffreys, who will speak at a tea meeting, and an address from Dr. P. Neumann on "The Mathematical Analysis of "1066 and all That" " will be the first meeting of the academic year. There will be a Careers Meeting as usual and also a visit to Oxford.

It is hoped that this year's programme will cater for all tastes. Suggestions for any change in the activities, or for speakers for future years would be most welcome; a book is kept in the Arts School for this purpose.

Simon Anscombe

A CRITICISM OF THE FOOTBALL LEAGUE EIGENVECTOR

D. R. WOODALL

Dr. A. N. Walker has made the following criticism of the eigenvector method of compiling an 'order of merit' after a tournament. He points out that the method only takes account of the team's wins, and not of its losses. Thus a team receives great credit for a win against a good side, but is not penalised for losing to a very bad side. If two teams A and B end up with the same number of points after an 'all play all' tournament, and if team A obtained its points against better teams (on average) than team B, it follows that team A must have lost to worse teams (on average). The team that beats the better opponents will benefit by scoring a larger number of points, unless it also loses to some worse opponents.

Because of this, Dr. Walker maintains that in an 'all-play-all' tournament the first order scores (sums of points scored) already take account of the quality of the teams beaten and so provide the fairest possible order. It is for this reason that in tournaments based on, for example, the 'Swiss system', in which not everyone plays everyone else, the usual method of separating ties is to credit a player with the sum of the scores, not of the people he beats, but of all the people he plays against.

The difficulty in the eigenvector method becomes particularly apparent if one considers an 'all-play-all once' tournament, with each team having a technical draw against itself, and with the usual (0-1-2) method of scoring. Then the i'th team's second order score is equal to the sum of *all* the first order scores, *plus* the scores of those teams that team i beats, *minus* the scores of those teams to which team i loses. In other words, team i is penalized more for losing to a good side than for losing to a poor side. To correct this, one should really subtract, not the score $p_j(1)$ of each team j that beats team i, but $C - p_j(1)$ for some constant C $\max p_j(1)$, C presumably being taken equal to the maximum possible number of points that a team can score. Unfortunately this is no longer a simple eigenvector method, but the first convergent could be used to separate ties.

To some extent a person's assessment of methods of scoring must depend on his psychology. A mathematician would tend to regard a draw as a neutral result, a loss as a negative result and a win as a positive result. He would probably expect the method of scoring to reflect this. In particular, he would expect that if all the results in the tournament were reversed, and the 'order of merit was calculated again, then the exact reverse order would be obtained. Of the methods discussed, only the eigenvector method does not have this property. The '0-1-2' method of scoring, however, tends to encourage people to think of a loss as a neutral result, a draw as a positive result and a win as a better positive result. This, and the eigenvector method, may accord better with the average person's tendency to forget the bad results and only take account of the good ones.

Whatever its disadvantages in this application, however, the eigenvector method certainly has its uses – in sociology for example, in finding the most influential person in a group. Here one wishes to find the person who wields the greatest overall influence in the group, and he is none the less influential by virtue of the fact that someone has influence over him.

THE OPTIMAL SIZE OF AN ORGANIZATION

I. J. GOOD

1. I was interested to read "The Optimum Size for an Establishment" in Eureka, October 1970, because in 1963 I wrote an article with the present similar title but never submitted it for publication. Since the two articles are complementary, I thought it would be worthwhile to hoist this one along with Petard's.

2. An important class of administrative problems is concerned with the choice of the size of an organization whose purpose is research or development or both. It would be hopeless to try to construct a universal model for optimizing the size, but the following simple class of models brings out some qualitative points and might sometimes be an adequate approximation.

3. Suppose the organization has an aim than can either be achieved or not achieved: for example, to find the cure for a specified disease. Although the value of success might seem intangible, bounds on it must be implicitly judged in terms of money. For otherwise there would be no basis for decisions to spend at least so much, and not more than so much. Let us suppose that the expected value of achieving the aim decreases with the time, T, for completion. This assumption is nearly always true. A wide class of reasonable decreasing functions will be approximated by the generalized exponential-decay forumla

$$V(T) = V = V_0 \exp(-aT^{\beta}) \quad (a>0, \beta>0),$$

where V is the expected value if the job take time T. Next we need a formula for the expected cost, C(T) = C, of achieving the aim in time T. A reasonable form for some kinds of organization is $C(T) = C = \gamma + \delta/T^{\epsilon}$ $(\gamma>0, \delta>0, \epsilon>0)$.

Fig. 1

Type I

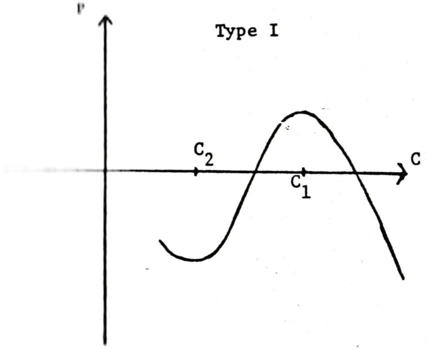


Fig. 2

Type II

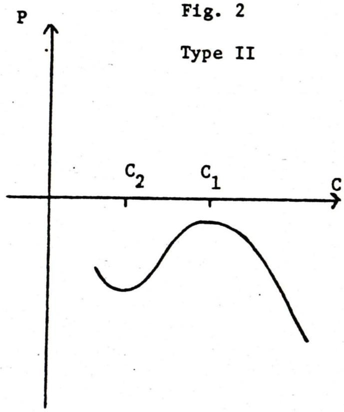


Fig. 3

Type III

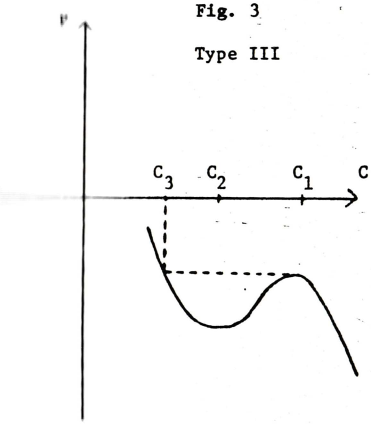
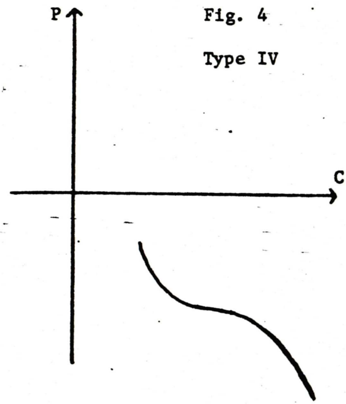


Fig. 4

Type IV



If the organization is of such a size that the aim will be achieved in time T, then the expected profit is

$$\beta V_o x^{1+\epsilon/\beta} = \alpha^{\epsilon/\beta}\delta\epsilon e^x$$

(Usually T would have at best, a subjective probability distribution for a given size of organization, but we ignore this complication.)
The expected profit is maximised when

$$P = V - C = V_o \exp(-\alpha T^\beta) - \gamma - \delta/T^\epsilon$$

where $x = \alpha T^\beta$. This equation always has either 0, 1 or 2 positive roots, the case of one root being very exceptional and not worth considering. If the roots are x_1 and x_2, where $x_1 < x_2$, we denote the corresponding values of C by C_1 and C_2. The graph of P as a function of C will take one of the forms shown in Figures 1 to 4. (The scales of the x and y axes are not equal). We call the corresponding problems those of Types I to IV respectively and also refer, with slight looseness of expression, to organizations of Types I to IV.

For organizations of Types I, II or III, there are two roots, but only organizations of Type I should be inaugurated, since they are the only type for which the expected profit can be positive. For Type II, if anything is to be spent at all, then the amount C_1 will be the least unprofitable in expectation. For organizations of Type III, if an amount less than C_3 has been legally committed, then the project should be cancelled, where $P(C_3) = P(C_1)$ $(C_3 < C_1)$. For Type IV, the less spent the better: such organizations are stale and unprofitable. For Types I to III, there is a local pessimal amount, C_2 that can be spent: it is the T.C.E. or Typical Cheeseparer's Expenditure.

4. The fact that so simple a model leads to conclusions conformable with common sense suggests that this kind of model-building might often be applicable to real organizations. In each application it would be necessary to reconsider the functional forms of V(T) and C(T), and to guestimate the parameters. It would be advisable to see what effect the variation of the parameters within reasonable bounds would have on the conclusions. Unfortunately this variation of the parameters would sometimes affect the "type" of the problem!

5. Note that we have been concerned here only with the amount that should be *voted* for an organization. The corresponding problems for organizations already in existence are apt to be more complicated.

DIVISION OF A SQUARE INTO RECTANGLES

BY BLANCHE DESCARTES

Figure 1 shows a rectangle divided into 9 squares, R,S,T, . . . Z, of all different sizes. This dissection was discovered by Z. Morón (Przeglad Mat. Fiz. *3*, 1925, pp.152-153). Such dissections can be found quite simply and straightforwardly as follows. Draw *roughly* a figure like Fig. 1, which looks plausibly like a dissection of a rectangle into squares. Denote the unknown sides of the squares by letters; thus we could let r = the side of square R, s the side of square S and so on. In order that the squares must fit together properly certain linear relations must hold; thus r=t+u because squares T, U, lie directly below R, and r+u= s and so on. Altogether we get 8 independent homogeneous equations connecting 9 unknowns (or, in general, one fewer equation that the number of component squares). These equations can be solved, thus providing a suitable dissection. In Moron's rectangle a solution is r = 14, s = 18, t = 10, u = 4, and so on; these values can obviously be multiplied throughout by any constant.

At a casual glance, Fig. 1 looks like a square, but in fact its height is 33 units and its width only 32. Can one fill a square with squares of unequal sizes? This is a much more difficult problem. It was first solved by Roland P. Sprague (Mat. Zeitschrift, 45 1939, p.607) in a complicated way. T. H. Willcocks, of the Bank of England, found a much simpler solution (Canadian J. Math. 3, 1951,

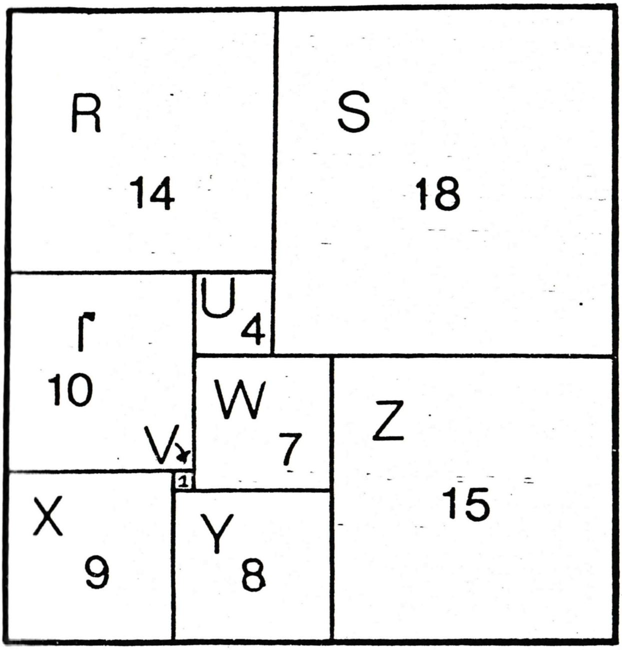


Fig. 1. Morón's rectangle

pp.304-308) using only 24 component squares; this solution is reproduced in several books on mathematical recreations (for example, Joseph S. Madachy's *Mathematics on Vacation* or Martin Gardner's *More Mathematical Puzzles and Diversions.*

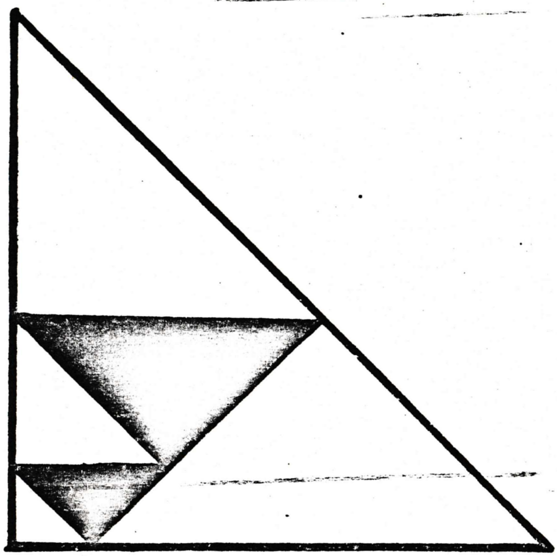
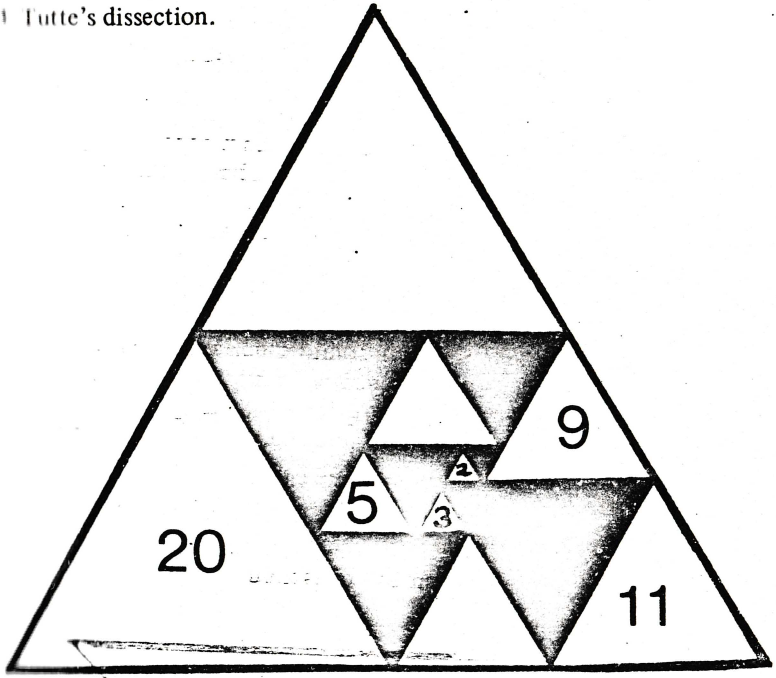


Fig. 2. Stone's dissection

Various mathematicians have tried to extend this result in various ways. For example, Arthur H. Stone succeeded in dividing an isosceles right-angled triangle into unequal triangles of the same shape (Fig.2.) William T. Tutte dissected an equilateral triangle into equilateral triangles (Fig.3.) If one makes a (not unreasonable) convention that triangles pointing upwards have positive sides, and those pointing downwards (coloured black in the figure) have negative sides, then this is a dissection into unequal triangles.

It recently occurred to me that there is another way of extending the idea. Instead of dividing a square into rectangles of different sizes but all of the same shape (namely squares), one can divide it into rectangles of different shapes but all of the same area. This leads to a system of nonlinear equations. The simplest solution is that shown in Fig. 4, a division of a square into 7 unlike rectangles J, K, L, M, N, P, Q. If we write respectively VJ and HJ for the vertical and horizontal sides of rectangle J, and similarly for the other rectangles, and let $h = \sqrt{19}$, the numerical values are .

Fig. 3 Tutte's dissection.



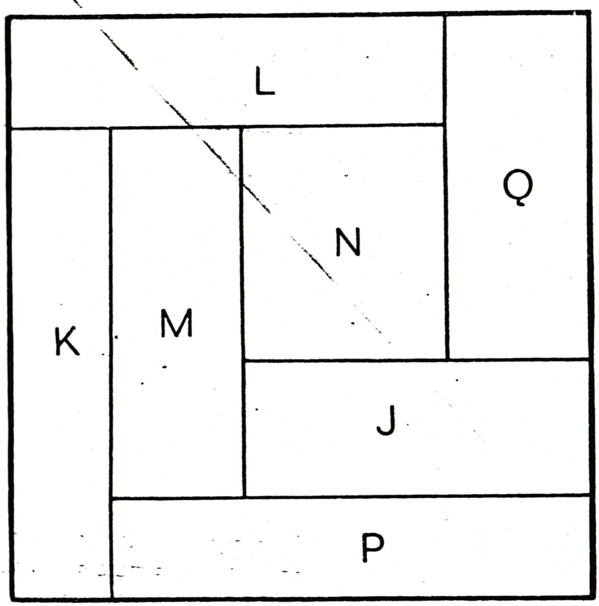


Fig. 4

VJ = 50	HJ = 126
VK = 15(7 + H) = 170.4	HK = 14(7-h) = 37.0
VL = 15(7 - h) = 39.6	HL = 14(7+h) = 159.0
VM = 25(h + 1) = 134.0	HM = 14(h-1) = 47.0
VN = 25(h - 1) = 84.0	HN = 14(h+1) = 75.0
VP = 10(8 - h) = 36.4	HP = 14(8+h) = 173.0
VQ = 10(8 + h) = 123.6	HQ = 14(8-h) = 51.0

The side of the square is 210, and the area of each of the component rectangles is therefore $210^2/7 = 6300$. It is easy to verify from the numerical values of the sides given above that all the rectangles do have this area and do fit together properly.

From this we can obtain a dissection of a square into (7+n) equal rectangles of different shapes, for any n>o. The construction is indicated in Fig.5 (for n = 5). Let a rectangle A be surrounded by rectangles B, C, D, E ... in a spiral arrangement in such a way that together they fill a unit square, and so that each of the "outer" rectangles B, C, D . . ., has area equal to 1/7 of the area of A. It is easy to find the sides of the rectangle satisfying these conditions; thus for n = 5, 7 + n = 12 we find the sides to be

VA = (8 x 10)/(9 x 11)	HA = (7 x 9 x 11)/(8 x 10 x 12)
VB = (8 x 10)/(9 x 11)	HB = (9 x 11)/(8 x 10 x 12)
VC = 10/(9 x 11)	HC = (9 x 11)/(10 x 12)
VD = 10/11	HD = 11/(10 x 12)
VE = 1/11	HE = 11/12
VF = 1	HF = 1/12

and similarly for other n. We now compress the square shown in Fig. 4 in suitable ratios vertically and horizontally so that it fits into the rectangle A; that is we reduce height in the ratio VA/210, and widths in the ratio HA/210. Notice that both of these are rational numbers. We now have a dissection of a square into (n + 7) rectangles, namely the n outer ones B, C,. . ., and the 7 inner ones, got from J, K, . . .Q by suitable compression; we may call these J*, K*,. . .,Q*. All these rectangles have the same area 1/(n + 7), and it would seem plausible that all these are of different shapes. But this has to be verified. Note that because of the equality of areas, it is enough to verify that the longer sides of the rectangles are all different.

Note first that the rectangle A is taller than it is wide, i.e. VA>HA. This is not at once obvious, but we have clearly

$$(8^2) \times (10^2) \times 12 > (7 \times 9) \times (9 \times 11) \times 11$$

and on division by (8 x 9 x 10 x 11 x 12) we get VA > HA.

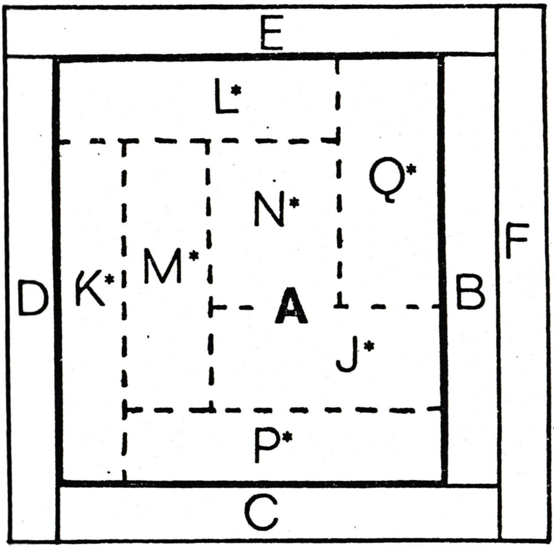


Fig. 5

In the same way we find that

$$VA = VB < HC < VD < HE < VF \qquad (1)$$

i.e. the longer sides of the outer rectangles B, C, D, E, F are in strictly ascending order. Since all the inner rectangles J* to Q* are strictly contained within A, and hence have longer sides strictly less than VA, it follows that no inner rectangle can have the same shape as any outer rectangle, and all the outer rectangles have different shapes. The only possible case of two rectangles of the same shape would therefore be two of the inner rectangles becoming so after compression. Notice that because the compression ratios are rational, this could only happen if the corresponding sizes before compression were in a rational ratio. From the values of the sides of the rectangles J, K,. . ., Q, this could only happen in 3 cases. Since VK: HL = 15: 14; we would get VK* = HL* provided the vertical compression factor is 14/15 times the horizontal one, i.e. provided VA:HA = 14: 15. But this is impossible, since VA > HA. For the same reason we cannot have VM* = HN*. We would conceivably get VP* = HQ* provided that we could have VA:HA = 14: 10. However, a calculation of VA:HA for successive values n = 0, 1, 2, 3,. . . gives

$$VA/HA = 1, 8/7, (8/7)x(8/9), (8/7)x(8/9)x(10/9),. . .$$

and all of these ≤8/7<14/10. Hence equality is in fact impossible, and the construction of Fig. 5 does divide a square into (n + 7) equal unequal rectangles.

THE MATHEMATICAL ASSOCIATION

PRESIDENT: B. T. BELLIS, M.A.
DANIEL STEWART COLLEGE, EDINBURGH 7.

The Mathematical Association was founded in 1871 (this being its centenary year) as the Association for the Improvement of Geometrical Teaching and aims not only at the promotion of its original object, but at bringing within its purview all branches of elementary mathematics.

The subscription arrangements are currently being reviewed but at present they are: full membership £3.60 junior membership 52½p.

The journal of the Association is "The Mathematical Gazette" and is published 4 times a year dealing with a variety of topics. The present editor is Dr. E. A. Maxwell.

AGGRO!

A demonstration is said to be *successful* by a certain R. Maudling (who shall remain anonymous) if over 40% of the people present are arrested. The rate of arrests at any time is proportional to the square of the number of unarrested people at that time, and is completely independent of the behaviour of the demonstrators. One week, 1000 people turn up to a demonstration, and 200 are arrested in the first hour. The next week, Chief Inspector Knacker expects 2000 people to turn up. How long will the demonstration have to last before it becomes successful?

Using only Hilbert's axioms 2.1, 2.2, 2.3, 3.1, 4.2, and 6.1, Playfair's Axiom, Pasch's Axiom and the Axiom of Choice prove that

"All triangles have three sides"

is a semi-simple locally compact monothetic quasitropic pseudo-sentence. State clearly the metalogical rules of inference used. (You are not required to prove the truth of the statement).

ON BADLY BEHAVED FISH FINGERS

BY D. R. WOODALL

'The piece of cod, which passeth all understanding.' (sic)

Suppose we are given n fish fingers, $ff_1, ff_2, \ldots, ff_n$, and a frying pan over a gas stove that we will assume to contain n discrete positions $P_1, P_2, \ldots, P_n$ in which fish fingers may be placed, possibly at different temperatures. Let $c = c(ff_i)$ denote the state of cooked-ness of a fish finger. A fish finger is *uncooked* if $c = 0$, and *exactly cooked* if $c = C$ for some fixed $C>0$, say $C = 1$. An uncooked fish finger placed in position P_i for time t achieves a state of cooked-ness $c_i(t)$ (independent of which fish finger it is), where the $c_i(t)$ are continuous and strictly increasing functions of t such that $c_i(0) = 0$ and $c_i(t) \to \infty$ as $t \to \infty$ so that in each position in the frying pan a fish finger cooks in a finite time. We assume that the rate of cooking of a fish finger is independent of its past history: i.e. a finger whose state of cooked-ness is c, placed in position P_i for time t, achieves a state of cooked-ness equal to $c_i(t + c_i^{-1}(c))$. The problem is this. Do there necessarily exist a cyclic permutation σ of $\{1,2,\ldots,n\}$ and time intervals $t_1, t_2, \ldots, t_n \geq 0$ such that, if the fish fingers are all placed in the pan uncooked at time $t = 0$ and the permutation σ is applied to them at successive intervals of $t_1, t_2, \ldots, t_{m-1}$, then they will all be exactly cooked at time $T := t_1 + t_2 + \ldots + t_n$?

This started as a practical problem; but I got fed up with having to eat my experimental errors, and so decided on a preliminary theoretical investigation. The answer is clearly 'yes' if n = 1 or 2 (by a trivial application of the intermediate value theorem), or if, for example, the functions are linear. (In general the number of time intervals could not be reduced, since, if the fish fingers cooked much more quickly in one position than in any other, it would clearly be necessary for each of the n fingers to get in the hot spot some time.) My initial conjecture was that the result would always be true for *all* cyclic permutations σ, but this turned out to be surprisingly difficult to prove – which is not surprising, since it is false. The first hint of this came when Dr. A. N. Walker produced a counter-example with four fish fingers (a modified version of which is exhibited below). However, Dr. Walker's fingers were in an obvious sense badly behaved (if not downright fishy), in that a finger in one of the positions would cook very much more slowly than in another position up to a certain critical cooked-ness, but would then suddenly begin to cook much faster. Dr. Walker suggested that a *well behaved* fish finger should be one that always cooks at least as fast in a hotter position as in a cooler one; i.e. one that satisfies $c_i \geq c_j$ whenever $i>j$ (after a suitable re-ordering of the functions), where $c_i \geq c_j$ means that

$$c_i(t + c_i^{-1}(c)) \geq c_j(t + c_j^{-1}(c))$$

for all c and $t \geq 0$. This seems an eminently reasonable condition to impose (although Professor Burgess has convinced himself that, in a corrugated iron frying pan, a fish finger in a warm trough could cook initially faster, but ultimately more slowly, than one on a hot hump).

If the fish fingers are well behaved, we are certainly assured of the existence of at least one permutation σ with the required property, namely the permutation $(123\ldots n)$. For suppose that ff_i starts in position P_i for each i $(i = 1,2,\ldots,n)$; and let $C_i = C_i(t_1,t_2,\ldots,t_n)$ be the cooked-ness of ff_i after time T, when the permutation σ has been applied after successive intervals of $t_1,t_2,\ldots,t_{n-1}$. Let E be the closed connected subset of R^{n-1} consisting of those points $(t_1,t_2\ldots,t_{n-1})$, with $t_i \geq 0$ for each i, for which there exists a $t_n \geq 0$ such that $C_1(t_1,t_2,\ldots,t_n) = 1$. With this (unique) value of t_n, the functions C_i are well defined and continuous on E $(i = 2,3,\ldots n)$. Within E, each t_i varies between 0 and $c_i^{-1}(1)$ $(i = 1,2,\ldots n)$. If $t_i = 0$, then $C_{n-i+2}\ \ C_{n-i+1}$ (reducing suffices modulo n if necessary), since, in every time interval except t_i, ff_{n-i+2} is in at least as hot a position as ff_{n-i+1}. Similarly, if $t_i = c_i^{-1}(1)$, then $C_{n-i+2}\ \ C_{n-i+1}$. We wish to prove the existence of a point in E at which $C_{n-i+2} = C_{n-i+1}$ for all i simultaneously, for at such a point all the C_1 are equal to C_1, = 1. One way of doing this is to extend the functions to the box.

$$F := (t_1,t_2,\ldots,t_{n-1}) : 0 \leq t_i \leq c_i^{-1}(1) \text{ for each } i$$

in R^{n-1}, in such a way that these inequalities still hold, with $C_2 > C_1$ on the whole of F E; (this is the most difficult part of the proof). If we put $f_i := C_{n-i+1} - C_{n-i+2}$ $(i = 1,2,\ldots,n-1)$, the result now follows immediately from the $(n-1)$-dimensional form of the following theorem.

THEOREM (n-dimensional intermediate value theorem). Let $f_1,f_2,\ldots,f_n$ be continuous real-valued functions defined on the non-empty closed set

$F := (x_1,x_2,\ldots,x_n) : a_i \leq x_i \leq b_i$ for each i $(i = 1,2,\ldots,n)$ in R^n. Suppose there exist real numbers $c_1,c_2,\ldots,c_n$ such that $\sup\limits_{x_i=a_i} f_i \leq c_i \leq \inf\limits_{x_i=b_i} f_i$ for each i $(i = 1,2,\ldots,n)$.

Then there exists a point $(x_1,x_2,\ldots,x_n)$ in F at which $f_i(x_1,x_2,\ldots,x_n) = c_i$ for each i $(i = 1,2,\ldots,n)$. (I have not found this result anywhere, but the proof, using Brouwer's fixed point theorem, is straightforward and is left to the reader (as usual!).

In the above proof, that the permutation mentioned always works, no comparison is made between the functions in different time intervals. Thus we could change the functions each time we move the fish fingers, provided that in each time interval the n functions are correctly ordered, and the argument would still work. This means that, given n well behaved fish fingers and any n frying-pans, we can cook the fish fingers to perfection in n time intervals by moving them round into a different frying-pan each time. (I have not tried this experiment.)

It is clear that the above argument works equally well for the permutation, n n 1 n-2 . . . 2). However, if $n>3$ the result is not necessarily true for *every* permutation σ, even if the fish fingers are well behaved, as is shown by the following modified version of Dr. Walker's example with $n = 4$. We take $c_1(t) :=$ $c_3(t) := t$, $c_4(t) := t\sqrt{2}$, and

$$c_2(t) := \begin{cases} \epsilon t & \text{if } t \leq \frac{1}{4\epsilon} \\ \frac{1}{4} + (t - \frac{1}{4\epsilon}) & \text{if } t \geq \frac{1}{4\epsilon} \end{cases}$$

for some sufficiently small ϵ (say $\epsilon := 10^{-6}$); and the permutation is (1342). It is not difficult to prove that there is no solution with $t_2 \geqslant 0$. The argument is helped by the fact that the equations for ff_1, $_2$ and $_3$ uniquely determine t_1 (up to order ϵ) – an accident, due to the fact that $(\sqrt{2})^2 = 2$ – and so by leaving ff_4 to the end we can get away with only one equation for each ff_1 (A diagram helps.)

To return to the badly behaved fish fingers – out of the frying-pan into the fire – it is not now true that there is necessarily *any* cyclic permutation o for which the result holds. Indeed, with Professor Burgess's help, I have concocted some perfectly pathalogical fish fingers that can't be cooked anyhow in three sittings, even with the aid of two different permutations σ_1 and σ_2, not assumed cyclic. The cooking functions are as follows.

$$c_1(t) := 2t/\epsilon$$

$$c_2(t) := c_3 := \epsilon t \qquad \text{if } t \leq 1/2\epsilon;$$

$$c_2(t) := \tfrac{1}{2} + (1+\delta)(t - 1/2\epsilon) \qquad \text{if } t > 1/2\epsilon.$$

$$c_3(t) := \tfrac{1}{2} + (1-\delta)(t - 1/2\epsilon) \qquad \text{if } t > 1/2\epsilon;$$

where ϵ is sufficiently small, say $\epsilon := 10^{-6}$, and $\delta := 10^{-1}$ (say). There are essentially only two cases to consider: that in which each fish finger spends one time interval on c_1, and that in which one of them spends all the time on c_2 and c_3. It helps in the elimination to use e, say, to denote an arbitrary number in the range $[1-\delta, 1+\delta]$, as there is only one case in which it is important that $1-\delta \neq 1+\delta$. The details are somewhat unpleasant and highly repetitive (like the fish fingers?).

In view of the extreme difficulty in cooking badly behaved fish fingers it is obviously essential that all fish fingers sold on the market should be well behaved. But until this can be guaranteed, there remains one problem of vital importance to the Fish Fryer's Association. This is to determine the minimum number $f(n)$ such that, given any n fish fingers and any frying-pan, one can guarantee to be able to cook the fingers exactly in at most $f(n)$ successive time intervals with the aid of $f(n)-1$ suitably chosen permutations. Clearly $f(1) = 1$ and $f(2) = 2$, but we have just seen that $f(3)>3$. Is $f(n) \leqslant 2^{n-1}$? Is it even true that $f(n)$ must be finite? Send your solutions – on a fish finger, please – the Department of Mathematics at Nottingham University. (C.O.D. not accepted.)

Magic Square

8		11		19
17		25		3
1		9		12

Complete the above magic square.

SOLUTIONS TO PROBLEMS

(The editor's thanks go to the compilers of this year's problems drive, Andrew Ellis and Colin Moore. He also wishes to thank Cliff Hones who stepped in at the last minute to solve the problems and without whom the editor's pencil would have been chewed to a stub by now, trying to solve the problems for publication.)

R.B.211. Critical path is : a, d, e, q, n or f, n, m or 1, m, p, r.
Engine hijacking plan must be ready by 1/9/74.

Demonstration must last at least 2hrs 40mins.

5 Queens: In the positions 4E, 5G, 6D, 7F, 8H, 115 squares are either occupied or attacked.

Magic Square: one solution is:-

8	5	11	22	19
24	16	2	13	10
17	14	25	6	3
15	7	18	4	21
1	23	9	20	12

well known phrases:

(i) $\forall \epsilon > 0 \; \exists \delta > 0$ s.t. $|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \epsilon$

(ii) $\forall \epsilon > 0 \; \forall x \in [a, b] \; \exists N$ s.t. $\forall m,n, \; m > N, n > N, |fm(x) - fn(x)| < \epsilon$

Honey magic square: just in case anyone has not spotted it, there clearly cannot be a solution.

Square numbers:

$$\begin{aligned} M &= 4^2 + 9^2 = 97 \\ M + 1 &= 7^2 + 7^2 = 98 \\ M + 3 &= 8^2 + 6^2 = 100 \\ M + 4 &= 10^2 + 1^2 = 101 \end{aligned}$$

BOOK REVIEWS

A COMPREHENSIVE TEXTBOOK OF CLASSICAL MATHEMATICS

BY H. B. GRIFFITHS AND P. J. HILTON (VAN NOSTRAND REINHOLD)

It is difficult to know what type of reader the book is aimed at – with over 600 pages, an enthusiastic one certainly. In Part II the authors seem to go beyond their declared intentions of making the book an introduction to the rigours of university mathematics. With somewhat muddled justification – "For instance how does one choose an electron from a pair of electrons?" – they introduce the axiom of choice, get through Zorn's lemma, Boolean algebra, and the propositional calculus, paving the way for the grand finale, on logic. The sections on algebra and arithmetic are the poorest of all. With such a plethora of fine examples on these subjects which illustrate the inherent beauty of mathematics one would have ~~thought that these sections would have~~ provided the greatest appeal of all. In fact major theorems are proved badly giving no clue to their wide application. During a lengthy introduction the authors philosophise on their intentions in writing the book, that their "leitmotiv is the very unity of mathematics". But surely the best way to illustrate this was not to cram as much mathematics into it, but perhaps to take a few abstractions and show their diversity of application.

-J. P. Conlon

COMPUTERS, CHESS, AND LONG-RANGE PLANNING BY M. M. BOTVINNIK

PUBLISHED BY SPRINGER-VERLAG, ENGLISH EDITION BY LONGMAN'S

The original title of the book was "An Algorithm For Chess," but, as the translator points out. "The audience should include not only chess players and computer theoreticians, but also students of management science, managers, psychologists, and the broad class of readers interested in the reach of the human mind." This stretches matters a little far. The book serves only as a general education rather than vocational instruction for the above groups of people. More illuminating is Botvinnik's comment, "Chess presents an inexact problem, with a large number of possibilities. Although the number of possibilities is large, it is theoretically finite; the finiteness however is irrelevant, since chess is played as though the number of possibilities were infinite." Therein lies the relevance of this book to long-range planning – the number of possiblities is also practically infinite, and we have to make decisions on less than complete information or analysis. A chess program thus is a useful prototype problem.

In putting forward his algorithm, Botvinnik makes two important contributions. Beginning with the concepts of attack and defence, his first contribution is a sequence of formulae describing the value and feasibility of a given attack. Thus his algorithm begins "what is there to try for?" and then continues "what happens is we make the attempt?" The next contribution he makes is to consider how to achieve this end. He says, "The reader here may press a claim against the author: is not the question stood on its head? Would it not have been better to construct the algorithm for the rules of the game and the moves of the pieces at the outset, and only then proceed to the logical algorithm? In other words, should not the positions of this and the previous chapter be reversed? No." And he convinces at least the reviewer that he has done it correctly. For having dealt with the attack-functions in the previous chapter, the technique of the moves of the pieces is dealt with not, as in previous programs one move at a time, but with the approach "How many moves will it take to move this piece to that square?" One is thus able to analyse further for a given effort.

But although the ideas that Botvinnik puts forward are excellent, the mathematical presentation of them is not. The formula

$$p_n = \tfrac{1}{2}\,(1 - n_{ik} - n_{rk} + a)$$

$$r_m = \tfrac{1}{2}\,(1 + m + o.1)$$

$$f = r_m + p_n - r_m p_n$$

bears little resemblance to the thought-processes of a human chessplayer. The FORTRAN code

```
LOGICAL F
F=N(I,K)+NA.LT.N(R,K) .OR. M.GE.O
```

not only gives the same result as the previous for all values occurring in the program, but is faster on the computer, and more closely resembles human thought. The introduction of complex numbers merely in order to keep separate count of Black men and White men is an unnecessary artefact, as is the concept of "intangible value of a piece", which serves only to clutter up the computer's store with the "body counts" claimed by each man on the board. The pretty diagrams drawn to illustrate the way to program the moves of the pieces does that task rather well, but their slavish copying to the computer's store has no advantage over a proper subroutine thought out afresh. However, we are told that Butenko, who is writing a Botvinnik chess program, has chosen to copy. Another unfortunate feature of the English edition is that the formulae have not been proofread.

One chapter is devoted to a demonstration that the algorithm would have discovered a combination the author once played, to beat Capablanca. The algorithm really works! In a preface, he mentions how the formula for evaluating an attack has missed out some possibilities. I would have preferred to see how the algorithm recognises double attack motifs such as the pin, the fork, the discovered attack, and the overworked piece.

On the whole, then, a valuable contribution to chess programming and to long range planning. But this book would benefit from a great deal of rewriting on the lines suggested above. Nor is it particularly cheap, at £2.50 for about 90 pages of text.

A. Iny.

ELEMENTARY DIFFERENTIAL EQUATIONS

BY DR. T. W. CHAUNDY (VAN NOSTRAND REINHOLD)

The first five or six chapters of this book deal with an excellent introduction to a course on Ordinary Differential Equations. Chaundy explains all the elementary tricks and manoeuvres in great detail and clarity, though perhaps at too much length. He then goes on to deal with series solutions and definite integrals. His treatment is thorough and rigorous and makes great use of the operator $\delta = x.d/dx$, an effective weapon many authors and lecturers omit. The later chapters are on the Hypergeometric functions, Legendre polynomials and Bessel functions, and a chapter on singular solutions. These show the author at his best with some very concise proofs involving awkward and cumbersome equations. This book gives a very full and detailed cover for about two courses on Ordinary Differential Equations, and I can recommend it to any mathematician starting the subject. It was edited by Dr. J. B. McLeod after the demise of Dr. Chaundy in 1966.

R. M. R. Brewis

ELEMENTS OF FUNCTIONAL ANALYSIS. VAN NOSTRAND REINHOLD

BY MESSRS BROWN AND PAGE

An introduction to functional analysis which is self-contained in that it justifies, rather than assumes knowledge, of any analytical methods required in the text. In the first half of the book, starting from a brief introduction to elementary analysis, it developes through the theory of metric spaces (perhaps useful for second year topology) to an outline of the theory of normed linear spaces and the concept of compactness. The second half of the book goes on to develop more fully certain aspects of functional analysis e.g. bounded linear functions and operators, the Frechet derivative and Baire's theorem and its application. The book is well laid out and the material prescribed in a manner easily understood without lacking rigour.

S. B. Dunnett

A SECOND COURSE IN MATHEMATICAL ANALYSIS

BY J. C. BURKILL AND H. BURKILL (C.U.P.)

This book is a fine analytic continuation of Analysis 1, (by J. C. Burkill, C.U.P.)and the two together cover all the analysisin Part 1A and the core of Part IB. In particular, this volume covers the move from the Euclidean plane to metric spaces, and gives an introduction to complex variables. The authors have obviously given much thought to the aim of the book, and have decided to concentrate on making this book and its predecessor a unified course in analysis for all mathematicians, rather than for intending specialists. For this reason, they have eschewed, correctly I think, such topics as topological spaces and Lebesgue integration.

It is difficult to find anything but praise for this book. Its clear lucid style and attractive presentation make it well readable as a textbook, and the treatment is almost invariably extremely good. Topics such as the Riemann-Stieltjes integral can seem like rehashes of the Riemann integral, but the authors have not fallen into this trap and have gone some way to pointing out the power of the Riemann-

Stieltjes integral, as well as giving motivation for the Lebesgue integral. (In passing there are a couple of errors in this section, pages 140 and 142). The section on complex variables is also excellent and proves that a good introduction to this topic can be given in a reasonably short space.

Finally, a word of praise for the notes at the ends of the chapters. These give a short history of the development of the subject, and also give the reader some idea of the ramifications and recommend source books.

G. S. Lucas

A COURSE OF GEOMETRY (C.U.P.)

BY D. PEDOE

This book, by a recognised master of the subject, is a fascinating account of geometry at the university level. It covers the Tripos geometry courses adequately and the author's insights give much valuable motivation for the theory of linear algebra.

The book contains a great deal of non-trivial exercises and is illustrated throughout with pertinent diagrams. The printing is up to the high standard expected from the C.U.P.
The only complaint one has is the price which is far beyond the undergraduate range. This is a great pity since the book contains so many new and refreshing ideas.

Paul Henry

THE ERGODIC THEORY OF MARKOV PROCESSES

BY SHAUL R. FOGUEL (VAN NOSTRAND REINHOLD)

This book is one of the Van Nostrand Series of Mathematical Studies, whose intention is to "provide a setting for experimental, heuristic and informal writing in mathematics that may be research or may be exposition". Written in concise, but largely clear, note-form, it deals with the asymptotic behaviour of the states of a given process and the existence of invariant measure, finite or sigma finite.

The principal results, supported by theorems and examples, are Hapf Decomposition, the Chacon-Ornstein Theorem, the Ergodic Theorem and the [illegible] convergence of Harris Processes.

A commendable feature is the chapter of definitions, notation and examples, which avoids the confusion to which some of the more complex passages may be prone. Clarity might also have been improved by greater detail in places: nevertheless this work, which pre-supposes a knowledge of elementary Functional Analysis and Real Variables or Measure Theory, and is therefore geared more to post-graduate requirements, is very good value at £1.25.

The Hon. C. W. Monckton

THE THEORY OF JETS IN AN IDEAL FLUID. BY M. I. GUREVICH. (TRANSLATED FROM THE 1961 MOSCOW EDITION BY R. E. HUNT AND EDITED BY E. E. JONES AND G. POWER.) PERGAMON PRESS, 1966.

Gurevich's book is largely concerned with the application of complex variable methods to the plane potential flows with free streamlines is used as models of jet and cavity flows in hydrodynamics. The author "has attempted to give a systematic account of the modern theory of jets". The text is heavily biased toward applications, and will be primarily of interest to fluid dynamicists and engineers. However, there are also some sections of considerable mathematical interest.

The first eight of the eleven chapters are concerned with steady two-dimensional irrotational flows of an inviscid incompressible fluid, the flow being such that free streamlines occur over parts of the fluid boundary. Chapter 1 is introductory. Chapter 2 is concerned with flows involving straight solid boundaries, which are solved by a combination of Schwarz-Christoffel and hodograph plane techniques. Chapters 3 and 4 discuss flows with curved solid boundaries. Of particular interest to mathematical readers of the book is the account of existence and uniqueness aspects of cavity flows past a curved obstacle (§ 18). This provided one of the earliest applications, by Leray, of the Leray-Schauder fixed point theory. Chapters 5, 6, 7 and 8 deal with a variety of specific problems in fluid dynamics where free surfaces are present. Unsteady flows are discussed in Chapter 9 and included here is the important problem of the surface impact of a wedge. Compressibility is introduced in Chapter 10, gravity and surface tension in Chapter 12. In Chapter 11 axisymmetric potential flows are considered. Here we no longer have the powerful tools of complex function theory, and the results are correspondingly less complete.

Understandably, the book has a natural bias toward Soviet contributions and, therefore, complements the surveys in Birkhoff & Zarantonello, "Jets, Wakes and Cavities"(1957) and in Gilbarg's article in "Handbuch der Physik"(1963) which show a bias toward Western writing. The reviewer is puzzled that there are translations of Gurevich's book. (The other is by a different translator and was published by Academic Press in 1965 at a higher price.) One translation, however, is very welcome for enabling English speaking research workers to bring themselves up to date with Soviet work.

Grant Keady

A GUIDE TO THE APPLICATION OF THE LAPLACE AND Z TRANSFORMS (2nd EDITION)

BY G. DOETSCH (VAN NOSTRAND REINHOLD)

This was written with the engineer or physicist in mind. The treatment is consequently less rigorous and much shorter and leaves several difficult proofs to the reader (poor engineer!) or for him to look up. An example of this is the sequence of rules governing Laplace Transforms in Chapter II, which is entirely proofless though references are given. The rules provide a basis for a much more thorough extension including several theorems and numerous worked examples. There is a long chapter on the Z-transform which includes much new material to the first edition, and includes some striking results. Though here again there is a lack of rigour. This book contains extensive tables of Laplace Transforms and makes good use of a caution sign in the margin for steps where engineers go wrong.

R. M. R. Brewis

MATRICES PURE & APPLIED   EDWARD ARNOLD LTD

BY [illegible] BRAND AND A. J. SHERLOCK

Although this book is intended as an extension of two earlier books – Matrices 1 and 2 – the first five chapters are devoted to an elementary investigation of matrix algebra, making the book suitable for use by A-level students with little knowledge of matrices. This section includes fairly extensive chapters on linear equations, powers of matrices and transformations. The "applied" section introduces Karnaugh maps and Boolean matrices in some detail, and concludes with a chapter entitled "Further Situations in which Matrices are used," amongst which are differential equations, networks, theory of games and knots; none of these latter topics is dealt with in detail, the examples serve more as an introduction to further reading.

R. Bowler

S.M.P. (SCHOOL MATHEMATICS PROJECT) BOOK E

This is the fifth of a series (S.M.P. Books A to H) of eight books which take the secondary school pupil as far as a C.S.E. examination level, the ground covered being the same as that for the corresponding S.M.P. O-level courses. Book E is thus of third or fourth year standard, and chapters on such "modern" topics as matrices, probability and networks, as well as traditional material like Pythagoras' Rule, the solution of equations, and the circle. One chapter I found particularly relevant was the brief so-called interlude dealing with some of the graphical misrepresentations so often employed in advertising. Initially the chapter on matrices seemed particularly lacking in examples, but the blocks of revision exercises made up for this to a large extent: for less able children however, I still feel that more and simple examples (which could be omitted by brighter pupils) would be very useful. On the whole the book seems to cover more than most pupils could cover in just a couple of terms – and one is loathed to omit any part of a course that interlocks as thoroughly. The visual presentation of this soft-backed book is pleasing with two-tone diagrams, and metric units are used throughout. The teachers' guide is noticeably thicker and contains the pupil's book, with its page numbers interleaved with comprehensive notes.

E. A. Workman

PRINCIPLES OF MATHEMATICS   MCGRAW-HILL

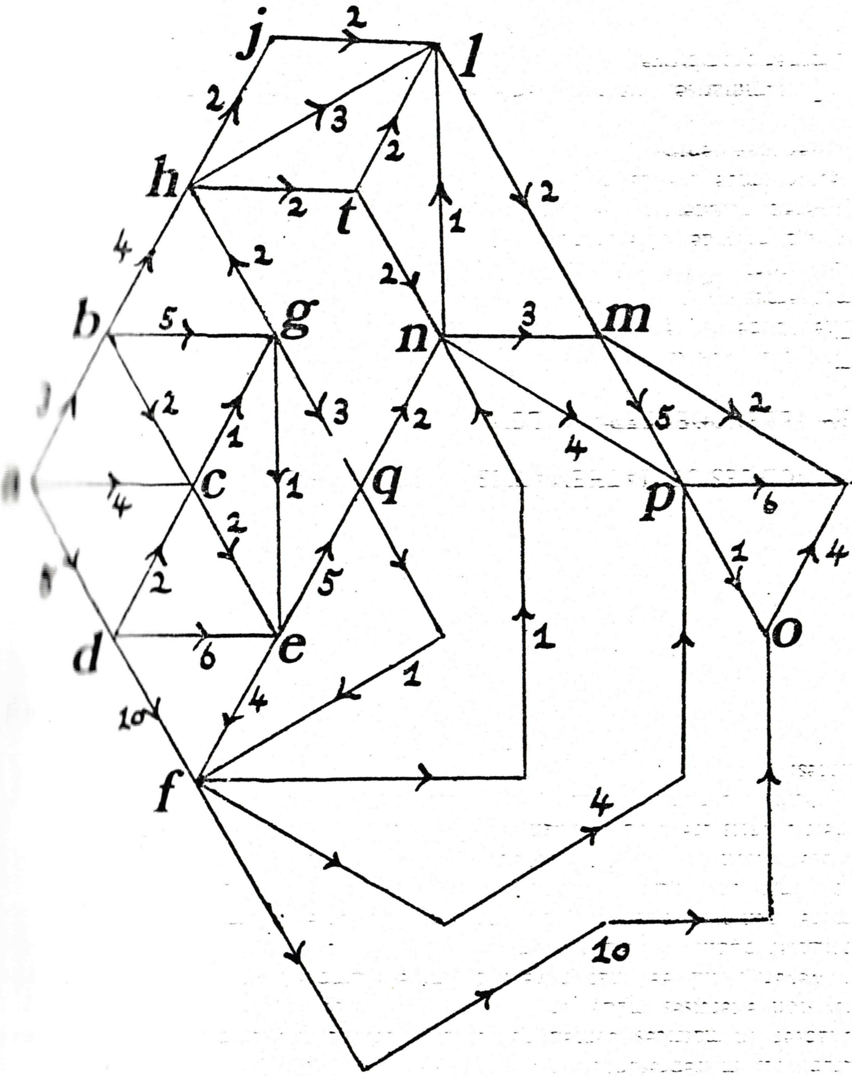
BY ALLENDOERFER-OAKLEY

This is a thick glossy American book, well laid out with plenty of illustrations. The scope is very wide with 15 chapters each on some aspect of modern mathematics. It was designed for first year undergraduates but in my opinion falls short of this level. It would be most useful if bought in the lower sixth and would provide a reference book for the next three years of study. Topics covered include sets, vectors, matrices, functions and the calculus. Subjects like vector space are defined but not developed, the emphasis being on breadth rather than depth.

To summarise, this book serves its purpose excellently; unfortunately its price is rather prohibitive.

C. J. Slinn

TriStar?



The above diagram has fallen into the hands of subversive agents: it represents the construction of an R.B.-211 engine from scratch (a), to finished product (r). The time in months taken by each stage in construction, and the independence of stages is indicated. What is the critical path? When should the agents have their engine hijacking plan ready by, assuming start of construction in January [illegible]