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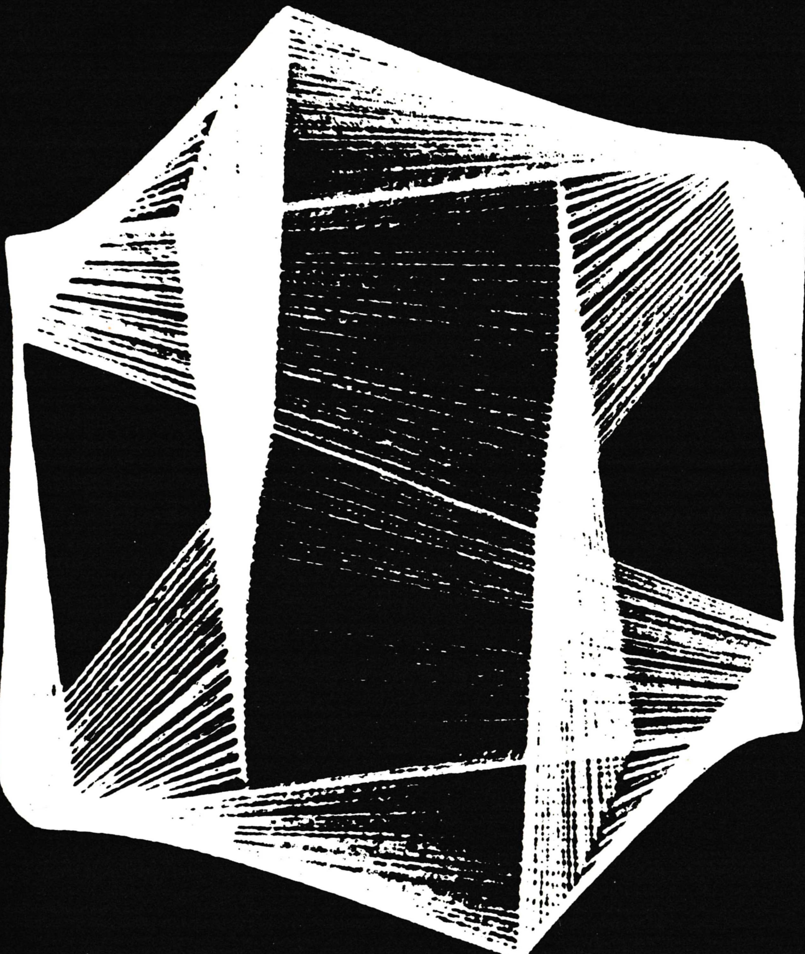

ROTATING A CUBE BY COMPUTER

No. 30 - OCTOBER 1967

two shillings and sixpence

EUREKA

The Journal of the Archimedeans, the Cambridge University Mathematical Society

Editor: C. J. Myerscough (Churchill)
Business Manager: J. J. Barrett (Churchill)

CONTENTS – No. 30, October 1967

	Page
Editorial	3
The first book of Moses	4
A Toroidal Twister	5
Beltrami or Force-Free Fields	6
Get Knotted!	8
Mathematics and the Physical Sciences	10
Networks and Squared Squares	14
Problems Drive	16
Rotating a cube by computer	18
Der Ring der Algebrungen	21
Incidence Incidents	22
Leave it to Aunt Minnie	27
The Paradox of the Surprise Examination	31
Problems Drive Answers	32
The Archimedeans	34
Mathematical Association	34
Book Reviews	35

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Editorial

University Teaching—a recent graduate's view

In theory, and often in practice, the system used for teaching mathematics in Cambridge is far better than that used at most other universities. The basic material which everyone must learn is covered in large lecture classes rather than in the small so-called seminars which are fashionable elsewhere; one lecturer does the work of several. On the other hand, students' own work is dealt with individually or in very small groups at supervisions. There they can move at their own pace, concentrating on the examination syllabus or investigating selected topics further according to their ability and inclination, and receiving help on their own particular problems. I shall attempt to describe what in my opinion are the two major difficulties encountered in the everyday operation of this system, and to suggest one innovation which could go a long way towards resolving them.

First, non-comprehension of lectures. Every time a lecturer fails to make most of a class understand a topic a huge amount of supervision time is wasted; perhaps a hundred supervisors each cover the ground again, and inevitably other problems are neglected.

Second, unsuitable supervisors. Supervisors who quite cheerfully admit that they know nothing whatsoever about the subject! Supervisors who don't admit this! Supervisors who never look at your work! Supervisors who set work on material not yet covered in lectures! Supervisors who sidetrack, and won't solve your problems!

But it seems rather unfair to blame only one of the two or three people at a supervision if it is a waste of time. Just as there are a few bad undergraduates, there are a few bad supervisors; all are soon recognized for what they are. Apart from these, any teacher will show an interest in his pupils' work provided they show an interest in it themselves. For example, a supervisor will not be encouraged to look at your work again if he finds that it consists of two sheets of mathematics and ten sheets of blank paper! He can hardly be expected to solve your problems if you have not made the effort of finding out what these are.

To remedy these two difficulties, we must improve communications, first between lecturers and undergraduates, and second between lecturers and supervisors. One way of doing both would be to issue duplicated lecture notes for all courses, preferably at the beginning of each term. Each student would have an accurate record of the lecture; the lecturer could concentrate on points of special difficulty, and on the relationships between results rather than on the mechanical details of their proof; an ample supply of exercises could be included. The work of writing the notes in the first place would help the lecturer to plan his course. Supervisors could refresh their memories easily, noting especially changes in approach and content since their undergraduate days. They could plan each term's work in parallel with the lecture courses. So the use of duplicated notes would benefit everyone—lecturers, supervisors, and undergraduates.

The Editor is very grateful to D. C. Lancashire, business manager for the eighteen months to last April, and to J. J. Barrett, who took over then. We have also received much help and advice from the following members of Churchill College: G. R. Farren, J. Filochowski, P. O. Gershon, R. H. Kavenagh, S. St. J. Wade, and D. H. D. Warren.

The Editor would like to officially deny that EUREKA receives financial aid from the C.I.A..

The First Book of Moses, commonly called Genesis,

in a new unauthorized rerevised non-standard version by King James I.V*

In the beginning, God created the heavens and the earth. And these were 2 creations.

So God, being an algebraist, created $\sqrt{2}$.

And He called his angels unto Him and demanded of them the value of it;

And they said unto Him 'Verily, Lord, it is ~ 1.414'.

Then Satan rose up and tempted Him, saying:

'If you are almighty, then command that this number be 1.41414141414 ....'.

But God cried aloud saying 'Get thee back to thy degenerate subspace! Such a thing must never be. For one thing, the next figure is a 2 anyway, but even if this were not so, this number which I have created cannot possibly be a recurring decimal, after any finite number of digits.'

Then Satan again tempted Him, saying: 'Command therefore that only recurring decimals shall exist'.

But God rebuked him and said: 'I may be almighty, but there are limits'.

And again Satan tempted Him a third time, with more subtlety, saying: 'Lord God, it is true, is it not, that this number cannot terminate after a finite number of digits, thereafter being a sequence of zeros'.

And God replied 'It is true'.

'But,' said Satan, 'may you not still amuse yourself in a finite sort of way by commanding that, having had one million rather disorderly places after the decimal point, the next million and one places shall be zeros, the sequence of digits being disorderly again thereafter?'

But God rebuked him a third time, saying: 'Behold, it is provable that, for any N, after N significant figures, it is impossible that the next N figures shall all be zeros.'

And lo! Satan, suitably crushed, took to the Klein bottle and retired into the nethermost regions of the complex plane. And St. Michael and all the angels did continue to evaluate the decimal expansion of $\sqrt{2}$ for ever and ever, world without end. *Amen.*

---

* This monarch was beheaded after —2 days on the throne—a deadunkind section (Ed.)

A Toroidal Twister

by I. N. Stewart

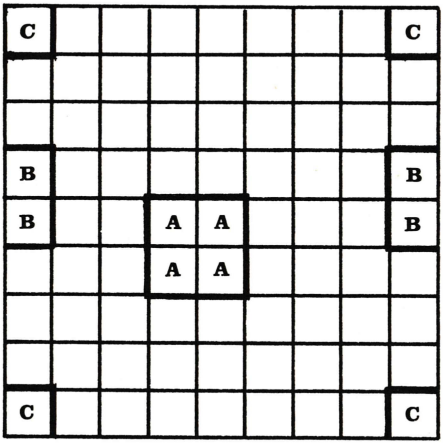


figure 1

1	1	0	1
0	0	0	1
1	0	0	0
1	0	1	1

figure 2

Consider a 9 by 9 square as shown in figure 1, with opposite edges considered as adjoining (thus giving a torus). It has 81 2 by 2 subsquares, e.g. A, B, C. We can put a digit 0, 1, or 2 in each cell of such a 2 by 2 square in $3^4 = 81$ ways. Put a digit 0, 1, or 2 in each cell of figure 1 so that each possible arrangement occurs exactly once in some 2 by 2 subsquare. For digits 0, 1 only, where there are 16 such 2 by 2 squares, a solution of the corresponding problem is given in figure 2.

Beltrami or Force – Free Fields

by V. C. A. Ferraro
Professor of Mathematics, Queen Mary College, University of London

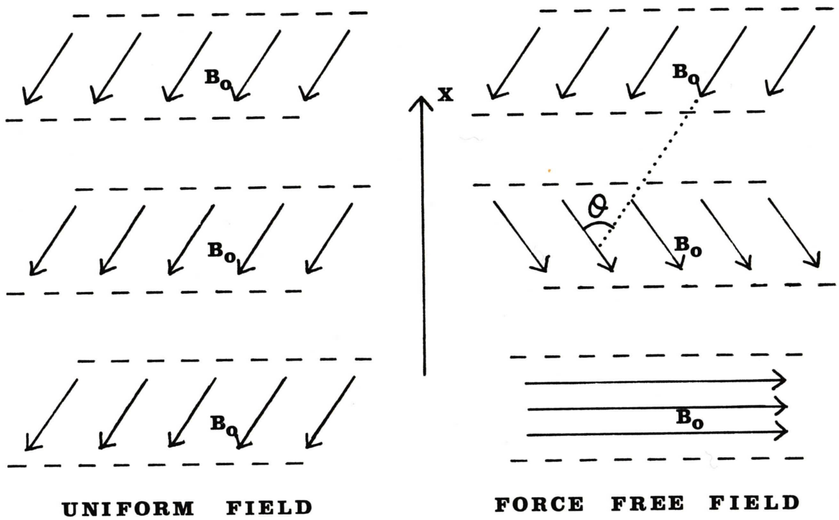


figure 1

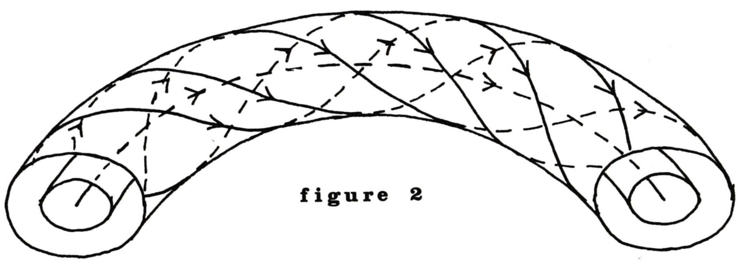


figure 2

Such fields were first studied by Beltrami in classical hydrodynamics; he considered whether a velocity field $\underline{v}$ exists such that the vorticity curl $\underline{v}$ at any point is in the direction of the velocity $\underline{v}$. In recent years the same fields have been introduced to discuss the equilibrium of the ionized material in the solar atmosphere subject to strong magnetic fields, where the gas pressure is insufficient to balance the force arising from the action of the magnetic field on the electric currents causing such fields. To avoid an impass, it was suggested that this mechanical force $\underline{J} \times \underline{B}$, where $\underline{J}$ is the electric current density and $\underline{B}$ the magnetic field, must vanish. Since $\underline{J} \propto$

curl $\underline{B}$ it follows that force-free fields satisfy the equations

$$(\text{curl } \underline{B}) \times \underline{B} = 0; \quad \text{div } \underline{B} = 0 \tag{1}$$

The existence of such fields has long been established but the fact that an electric current can flow in the <u>same</u> direction as the magnetic field to which it gives rise runs counter to our notion that the magnetic lines of force produced by a current flowing in a closed loop are closed curves which link the current loop. Yet it is possible to construct a simple example of such a field. In fact, if the lines of force of a uniform magnetic field lying in a series of parallel planes be rotated about an axis perpendicular to such planes through arbitrary angles, the resulting field is force-free, as can be easily verified. (Fig. 1)

If the planes are perpendicular to the axis of x, the uniform field B_0 parallel to the z-axis so transformed is the force-free field whose resolutes are 0, $B\sin\theta$, $B\cos\theta$, where θ (an arbitrary function of x) is the angle through which the field lines in each plane are turned.

There exists an interesting topological theorem relating to Beltrami fields; the theorem states that the lines of force of a Beltrami field confined within a region of space lie on a nest of tori.

To prove this theorem, we note that the first of equations (1) requires that

$$\text{curl } \underline{B} = \alpha \underline{B} \tag{2}$$

where α is a scalar function of position. From this it follows at once (since div curl $\underline{B} = 0$) that

$$\underline{B}.\text{grad } \alpha = 0 \tag{3}$$

i.e., the lines of force of the Beltrami field lie on the family of surfaces $\alpha =$ constant. Suppose now that the lines of force lie on a simply-connected region. Then the lines of force must form closed curves on these surfaces. Thus, if s denotes the arc length measured along such a closed curve Γ in the direction of the field $\underline{B}$, we have

$$0 < \oint_\Gamma \underline{B}.\underline{ds} = \int_\Sigma \text{curl } \underline{B}.\underline{dS}$$

using Stokes' theorem, and by (2) this gives $0 < \int_\Sigma \alpha\, \underline{B}.\underline{dS}$, where Σ is an open surface $\alpha =$ constant bounded by Γ. But since the lines of force lie on the surface Σ, the last integral vanishes. We have thus reached a contradiction and the assumption that the surfaces $\alpha =$ constant are simply connected is false. The lines of force must therefore lie on a nest of tori, one inside the other, with a limiting curve which is itself a line of force (Fig. 2). It will be noted that, in general, such lines of force will have no beginning or end.

Get Knotted!

by A. G. Smith

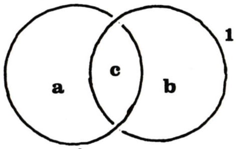


figure 1

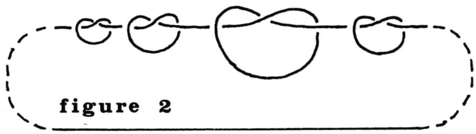


figure 2

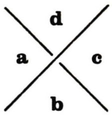


figure 3

figure 4

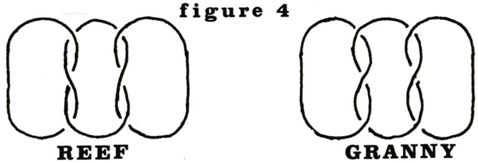


figure 5

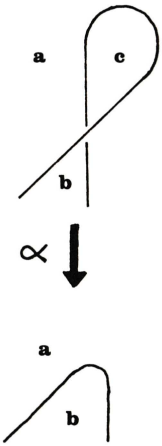


figure 6

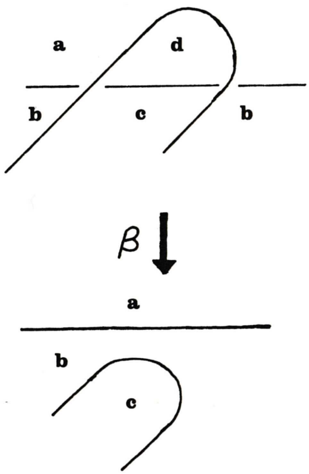
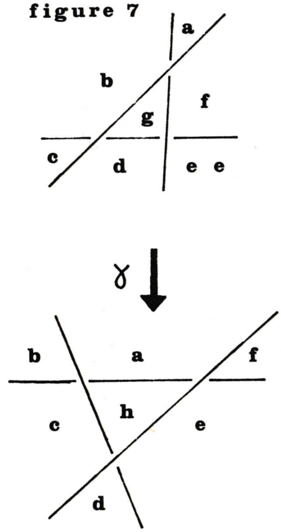


figure 7



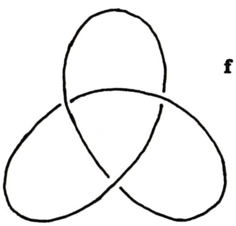
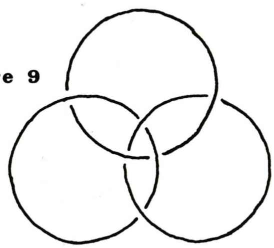


figure 8

figure 9


To many of us, no doubt, knots are merely peculiar things that Boy Scouts persist in doing to innocent pieces of string. However, the ever widening grasp of Pure Mathematics has fastened on even the humble knot, producing one of the few physically picturable branches of algebraic topology. I shall endeavour here to give a glimpse of the mathematical theory of knots.

As we shall be defining a concept of knottedness, and equivalence of knots, we must find some means of preventing our knot being illicitly untied—for else any two knots could be claimed to be equivalent since the first could be untied and the second then tied. The easiest way to eliminate untying is to identify the two loose ends to make a loop.

We thus make our definition of the knot: a knot is an embedding of the circle in Euclidean 3-Space. (Embedding has a technical sense, but it suffices to think of it in its intuitive sense of a 'putting in'). An n-link is similarly defined as an embedding of n circles in Euclidean 3-Space. These definitions do not exclude such monstrosities as figure 2. Such knots are aptly known as 'wild' knots; we restrict ourselves to 'tame' knots, which may be thought of as having only finitely many cross-overs.

Two knots or links will be equivalent if it is possible to continuously deform the whole space so that the one knot (or link) is carried into the other, and vice versa. We shall now ascribe to each knot or link a group. This is in fact the so called fundamental or Poincairé group of its complement, a standard topological invariant. Consider a picture of a knot or link in the plane with cross-overs indicated; such a picture is called a presentation of the knot or link (figure 1). Allot to each bounded region of the plane in the presentation a generator; allot the unit element 1 to the unbounded region. At each cross-over allot a relation between generators thus: Figure 3 gives $ab^{-1} = dc^{-1}$.

The perceptive reader may well have noticed that we have ascribed a group not to a knot but to a presentation of a knot. However, two presentations of a given knot can be transformed into each other by repeated applications of the operations α, β, and γ shown in figures 5, 6 & 7. If we consider what happens to the group in the three cases we see that:

In figure 5, the cross-over merely defined c in terms of a & b: $c = ab^{-1}a$. Thus the group is unaltered by α and α^{-1}.

In figure 6, before applying β, $d = cb^{-1}a = cb'^{-1}a \Rightarrow b = b'$; then, as above, the cross-over merely relate d explicitly to a, b & c, so that the group is unchanged, by β and by β^{-1}.

In figure 7, before applying γ, $g = f^{-1}ab = dc^{-1}b = de^{-1}f \Leftrightarrow fa^{-1} = dc^{-1}, c^{-1}b = e^{-1}f$, and $g = fa^{-1}b$; after applying γ, $h = ab^{-1}c = cd^{-1}e = af^{-1}c \iff af^{-1} = cd^{-1}, b^{-1}c = f^{-1}c$, and $h = af^{-1}c$. Thus, g and h depend explicitly on a, b, c, d, e, and f, and may be eliminated; the relations between a, b, c, d, e, and f are unaltered. Thus, γ and γ^{-1} do not alter the group.

We have now ascribed to any knot, or link, a group; knots with different groups are clearly distinct. It is less clear, and indeed false, that knots of the same group are equivalent; however it was shown in 1957 that a knot with group Z (the integers) is equivalent to the so-called unknot (i.e. it is itself unknotted).

We can now actually prove that a knot exists! Consider the simplest knot of all, with but three cross-overs, the 'trefoil', shown in figure 8. The group S has generators a, b, c, d and relations $a = bc = cd = db$. Eliminating a, and d, S has generators b, c such that $bcb = cbc$. Substituting $x = bc$, $y = b^2c$, we get $y^2 = x^3$; S is thus generated by x, y with this relation. Now define a homomorphic image of S by mapping x^3, y^2, and $xyxy$ to the unit; this maps S into G say, which is generated by p, q, with relations $p^3 = q^2 = 1$, $pq = q^{-1}p^{-1} = qp^2$. This is isomorphic to the permutation group on three elements, and so is non-abelian. Thus S is non-abelian, and the trefoil is knotted. Again, the knot of figure 1 has a group which is free abelian on two generators; however, two unlinked loops have a free group on two generators, with no relations, and in particular no commutativity relation. The configuration is thus linked.

The group of a knot is not to be thought of as an infallible weapon—it fails to distinguish between the reef knot and the granny, for example. These have been shown to be non-equivalent. (figure 4). Much of knot theory consists of finding more computable objects associated with the group—gaining in practical applicability at the expense of sharpness of the test.

I leave it to the reader the problem of finding whether Knotung (page 20) is knotted, and if so, whether it is equivalent to the Borromean Rings (figure 9), with which it shares the property that cutting any one loop releases the other two completely. (In fairness, I should admit that I cannot prove either, but it is intuitiviely obvious if one plays with bits of string.)

Mathematics and the Physical Sciences

by P. J. Bowler

When the modern mathematician thinks of the relationship between his own subject and the physical sciences, he often tends to assume an air of superiority, derived from the knowledge that the basis of the scientists' method always seems to involve the description of phenomena in terms of mathematics, and that without this method science, as we know it, would not exist. A feeling of superiority is, perhaps, not really appropriate when comparing two widely different disciplines such as experimental science and 'pure' mathematics, but it must be admitted that the former is committed to the use of a method in which the latter plays an essential part. Because of the success which has accompanied this procedure it is difficult for us to conceive of science in any other form. Yet when we consider its history, it soon becomes apparent that the relationship between the two is not as clear-cut as is sometimes assumed. The use of mathematics by the scientist involves presuppositions which have been described as metaphysical in character, and we shall attempt to show that this description is justified. Although metaphysics is regarded as rather a dirty word nowadays, the use of mathematics in describing nature does fit into such a category since by doing this, the scientist is imposing a product of his own mind on to the universe, rather than revealing something which is already there. It is this and the method by which it is done which separates the scientist from the mathematician so completely that superiority on either part is meaningless.

The belief that the universe is suceptible to mathematical treatment is by no means as obvious as it might seem. We know that before the so-called scientific revolution men were quite satisfied with a physics which was not constructed in this way and bitterly resented the introduction of another viewpoint. Even now, although we use mathematical descriptions, we are always sure that these are not completely accurate, and that sooner or later a more detailed study will reveal this and necessitate another attempt. If we believe, as do most philosophers of science, that we can never reach any ultimate truth in this field, we are faced with the prospect of a never ending increase in the complexity of our descriptions without these ever being complete. In this sense it might be said that the world is not truely representable in mathematical terms, owing to its extreme, possibly infinite, complexity. In the course of the last few centuries, scientists have been forced to learn that their aim must be to make closer and closer approximations to what really happens, not to lay down systems which are assumed to be complete. An example of the trouble caused by ignoring this is seen in the enormous faith placed in Newtonian mechanics in the 18th and 19th centuries and the consequent upheaval caused by the theory of relativity which showed the inadequacy of this system.

It must not be thought that the above arguments offer any criticism to the scientific method, since this has provided its own justification in the tremendous value of its results. What we are attempting to do is to show that in such studies as physics, the mathematical descriptions used are derived from our determination to see things in this way, rather than from some inherent property of the things themselves. As it has turned out, this is a very successful way of interpreting the phenomena we observe, but it is not the only way. The Aristotelianism which existed before the scientific revolution was not bad science, as has often been claimed, it was an entirely different way of looking at the world. It was both anthropocentric and anthropomorphic, and thus insofar as it dealt with physical problems, it demanded teleological explanations:- 'a stone falls to the ground because it seeks its proper, natural place'. In this scheme, a description of how such a fall occurred was to a great extent irrelevant. Hence there was little effort made to correct hasty decisions made on the basis of crude experience (e.g. that a heavy object fell more quickly than a lighter one). This was, in fact, a study of 'why' not of 'how' and it was the reversal of this emphasis which gave rise to modern science. At first there was too great a change and we find the 'mechanical philosophers' attempting to reduce everything to matter in motion and hence to an exact mathematical description. This idea had its successes, but also ran into some difficulties, since the new method required a close observation of its subject matter—unlike the previous discipline—and this soon revealed the inadequacies of its preconceptions. For somewhat different reasons, modern science has again demonstrated the limitations of the mathematical method, and we have now come to realise that our theories can never be more than approximations to the true workings of the universe. However, in the course of time it has been shown that the attempt to describe things in this way does produce valuable results and we must thus consider how this is possible if we are continually changing our ideas concerning the basic workings of nature.

There is one type of scientific theory which may truly be said to explain one set of facts in terms of another more basic or more familiar set, and this type seems to be affected to but a small extent by the great changes which take place in our fundamental ideas. An example of this is the Kinetic theory of gases, which explains the gas laws in terms of mechanics by postulating the existance of unobservable entities called molecules. It is now about half a century since the concept of an atom as an elastic particle was overthrown, yet the Kinetic theory still stands. This is because the new theories introduced concerning the structure of matter must always contain within them the original facts, however many new ones have since been discovered. Thus no theory has yet denied the existance of some sort of entity called a molecule, although the structure of this has been found to be progressively more complicated as time goes on.

The more fundamental type of theory appears to have no explanatory powers, but is concerned with correlating data and is actually describing how the universe works, rather than explaining anything. At any given time the currently accepted theory must bring together all the available facts, but since science is essentially concerned with observation and experiment, new facts are always coming to light. There is disagreement over the spirit in which scientists undertake this task, one school claiming that it is an attempt to extend the current theory as far as possible, the other that it is the desire to find where this breaks down, so that another, better attempt can be made. In any case, the number of phenomena to be explained grows, and eventually discrepancies arise and the theory has to be replaced. A once well established theory may still have uses after it has been superseded—by virtue of its original position it must have brought together a great number of facts and it may still be the easiest way of dealing with these facts, even if we regard it only as an approximation e.g. our continuing use of Newtonian mechanics even though relativity has shown that some of the basic presuppositions of this are false.

We thus see that for any given theory the mathematical descriptions involved are of the utmost importance—without this tool, the scientist would be unable to formulate his ideas, and his development of new theories depends upon the availability of sufficient mathematics. As we have shown, however, the scientist must always regard his theories with a certain amount of suspicion since they are always likely to be made redundant by the discovery of new facts. To the scientist, therefore, mathematics can never be more than one part of his work. His essential criterion must always be the relevance of his ideas to the world in which he makes his observations and this will also be the source of any inspiration he might have for a new theory. The unique combination of mathematics with experiment which has been created in the physical sciences has produced results of the greatest importance to all mankind, but it is essential that both sides become aware of the relationship between them. In this article we have attempted to point out the special way in which science uses the work of the mathematician. We hope that it will help to clear up misunderstandings and reduce the old feeling of rivalry which was mentioned above.

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Networks and Squared Squares

by G. H. Morley

A dissection of a rectangle into a finite number $N > 1$ of squares (called elements of the squaring) is a squared rectangle of order N. The term 'elements' is also used for the lengths of the sides of the elements. The Bouwkamp code[3,4] of a squaring lists the elements from left to right, working down, enclosing within brackets those elements whose upper sides lie on the same line. The squaring is perfect if no two squares are congruent. It is compound if some of its squares as arranged form a smaller squared rectangle; otherwise it is simple. Let l (m, n) be the smallest number of squares used in a perfect (simple, simple perfect) dissection of a square. It is known[4,8] that $m = 13$, whilst l and n are unknown: $20^4 \leqslant l \leqslant 24^{2,5}$ and $20^4 \leqslant n \leqslant 25$. The best simple perfect squared square (SPSS) published,[4,5] found by T. H. Willcocks of Bristol in 1959, is of order 37 and side 1, 947.

By means of the so-called rotor-stator method, developed by four members[1] of the Trinity Mathematical Society about 1938, it is possible to construct pairs of mutually congruent but differently squared rectangles, such that they can be oriented to share a common corner element. The addition of two extra squares gives a compound squared square which is generally perfect.

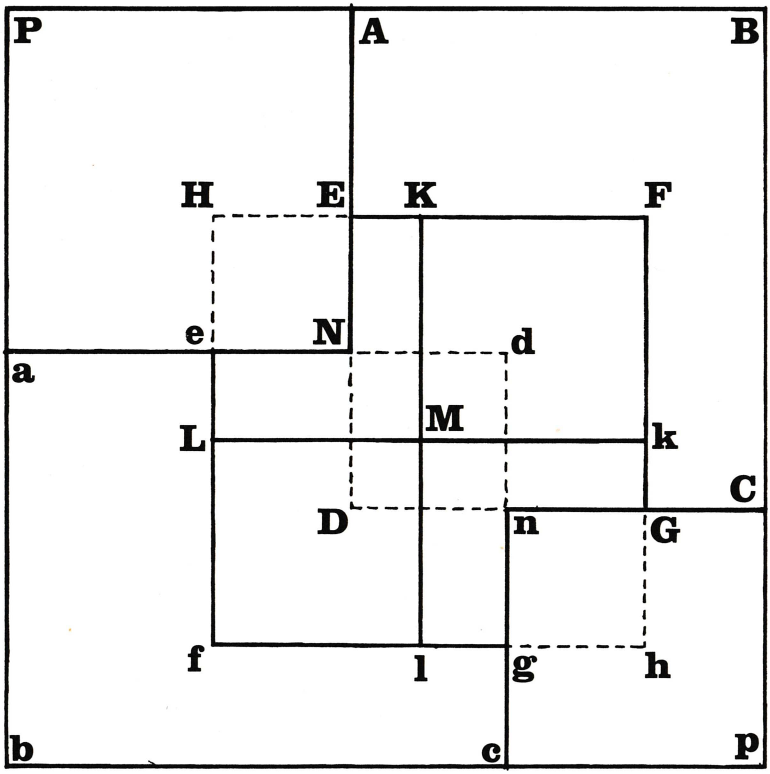
Few SPSS's were known until I devised a method of constructing an unlimited number of a particular kind. The smallest such solution to date has 60 squares and (reduced) sides 616, 467. The method is as follows:

We start off with the two rectangles HKML, hkMl in the diagram, which possess the properties just described: they are oriented as shown, so that the squares HENe, hGng are congruent. ABCD, abcd are two other rectangles with the above properties; the squares EFGD, efgd are congruent. Clearly, we can scale the two sections of the diagram so that they fit, as shown. Then the square is dissected into the squares of ABCGFE, abcgfe, LMKENe, and lMkGng, together with the squares PANa, pCnc, LMlf, and KMkF. In general this dissection is simple and perfect.

Corresponding to each squared rectangle is a planar network or net whose vertices $V_1, V_2, \ldots, V_n$ correspond to squares. Vertices V_a, V_b corresponding to sides of a simple rectangle are joined by an edge to give a c-net. The complexity C of a c-net is the number of its spanning trees. It has the remarkable property that when the semi-perimeter of the rectangle is scaled to equal C, every element is an integer. If each edge of a c-net is a wire of unit resistance, a simple squaring is obtained by placing an e.m.f. of value C in one of the wires. The elements are equal to the currents. Their highest common factor is the reduction factor of the squaring.

An n x n square matrix A has elements a_{ij} where $a_{ij} = -1$ or 0 for $i \neq j$, according as vertices V_i, V_j are connected or not. a_{ii} is the number of wires incident at V. It can be shown that the absolute value of all first cofactors of A equals the complexity of the c-net, and that the current flowing in V_cV_d is found by evaluating the determinant of the matrix got by eliminating from A rows a and b and columns c and d. The absolute value of any first cofactor of this matrix represents the number of distinct trees of the complete graph on n vertices; it is easily shown[6] to be n^{n-2}. This immediately proves Cayley's Theorem (Helmer's conjecture[7].)

In any squared square of side s in its reduced terms, choose elements a, b such that a-b and s are relatively prime. If the reduction factor R be identified by a subscript denoting which element is being squared, then we may prove that s is a factor of $(a-b)R_s$, and so of R_s. Hence the complexity of the c-net of a squared square always



has a large squared factor. This is also true of a rectangle 2 × 1, for which no simple perfect squaring is known. Finally I make the following conjecture: every squared rectangle with a reduction factor less than 4 is perfect.

I am particularly grateful to C. J. Bouwkamp, R. L. Brooks, A. J. W. Duijvestijn, P. J. Federico, C. A. B. Smith, W. T. Tutte and T. H. Willcocks for the information on squares that they have passed on to me during the past four years:

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Problems Drive 1967

set by A. G. Smith & P. N. Toye

(A) Extend the following sequences by at least three terms:

(1) 0, 0, 0, 0, 24, ...
(2) 0, 1, 2, 9, 44, ...
(3) 4, 6, 12, 18, 30, 42, ...
(4) 0. 5, 1, 3, 6, 12, ...

(B) Given two identical cubes and paint of four different colours, with sufficient paint of each colour to paint three faces of a cube, in how many distinct ways can the cubes be coloured, with the condition that no two adjacent faces have the same colour, and that the entire face of a cube must be painted with one colour, if

(i) all faces must be painted?

(ii) faces may remain unpainted?

(C) A lecturer from the faculty of Archaeology and Anthropology, who has an interest in mathematics, is investigating the traditional island whose natives belong to three tribes: the Damtps, who always tell lies (usually referred to as 'first-order approximations'), the Dpmms, who always tell the truth, and the Stats, who lie and tell the truth alternately. Also, they refuse to talk directly to strangers, and insist on talking through interpreters.

The lecturer (hereafter referred to as 'L'), trying to discover where he is going, meets three natives, whose tattoo-marks proclaim them to be from different tribes, and whose names (embroidered on their loin-cloths) are Artaxerxes, Bechstein, and Cadwallader (hereafter referred to as A, B, C, not respectively).

The following conversation took place:-

L(to B): What tribe does A belong to?
A: He say 'Dpmm.'
L(to C): What tribe does B belong to?
B: He say 'Stat.'

L(to A): What tribe does C belong to?
C: He say 'Damtp.'
L(to B): Where does this road go?
C: He say it leads through impenetrable jungle to Bhatmandu.*
A: He say it goes to Llanfairpwllgwyngyllgogerychwyrndrobwllllantysiliogogogoch.*

To which, if either, of these places did the road go?

(D) In three-dimensional space, there is more than one configuration of three loops such that no two are interlinked, but the three cannot be separated, one such configuration being the Borromean Rings (page 8, figure 9).

(1) How many distinct configurations are there with these properties?

(2) Draw (or describe) one such, distinct from the above and its mirror image. (Two configurations are distinct if they cannot be deformed into each other in 3-space).

(F) With the notation described below, for n = 2, 3, 4. . . . , find functions $f_n(x)$, defined for $-\infty < x \leqslant +\infty$, and taking values in this range, such that:-

(i) $f_n^{(k)}(x)$ is not identically equal to x, for $0 < k < n$,

(ii) $f_n^{(n)}(x) = x$, for all x.

Notation: $f^{(1)}(x) = f(x)$; $f^{(n+1)}(x) = f(f^{(n)}(x))$

(G) Cover the plane with congruent $(2n + 1)$-gons, with no angle equal to 180°.

(H) A philosopher is sitting beneath an apple tree. An apple at the top of the tree begins to fall, with initial velocity zero. It is to be assumed that:

(i) in time δt there is probability $1/20\ \delta t + 0(\delta t)$ of another apple being dislodged by a falling one,

(ii) in time δt there is probability $1/10\ \delta t + 0(\delta t)$ of a falling apple being caught up in the tree, independently of (i).

There are 10. 24 feet of tree for the apple to pass through. What is the expected number of apples which will emerge from the tree?

(1 is a large number. $g = 32$ ft./sec.^{2})

(I) A closed surface may be regarded as a plane polygon, with certain edges identified; in particular, the projective plane can be regarded as a square with edges identified so that the vertices are a, b, a, b in cyclic order. An M-triangulation of a closed surface V is a dissection of V into triangles, such that:

(i) at any edge exactly two triangles meet

(ii) any two triangles meet; and meet in either one vertex or one edge only.

M-triangulate the projective plane.

(J) n points are placed on the edge of a disc, and all joined by straight lines. Find into how many portions (at most) the disc is dissected by these lines, in the cases n = 1, 2, 3, 4, 5, 6.

(K) In the University of Grantaford there are four men's colleges, Michaelhouse, King's Hall, Angel Hall and Whewell House. There are also two ladies' colleges, Lady Bertha Hall and Newton Hall.

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* Two local villages.

Inter-college rivalry among the men is such that no man will cross the path of a man from another college; nevertheless the undergraduates from each of the men's colleges find it convenient or necessary to visit from time to time each of the ladies' colleges, the Lecture Rooms, and the Jolly Quadric (a local Public House).

Find paths which will allow them to do this.

(For the purposes of this question, you may assume that the surface of the Earth is of the form of a Klein bottle, which can be represented by a square with opposite edges identified with a 'twist' in one pair.)

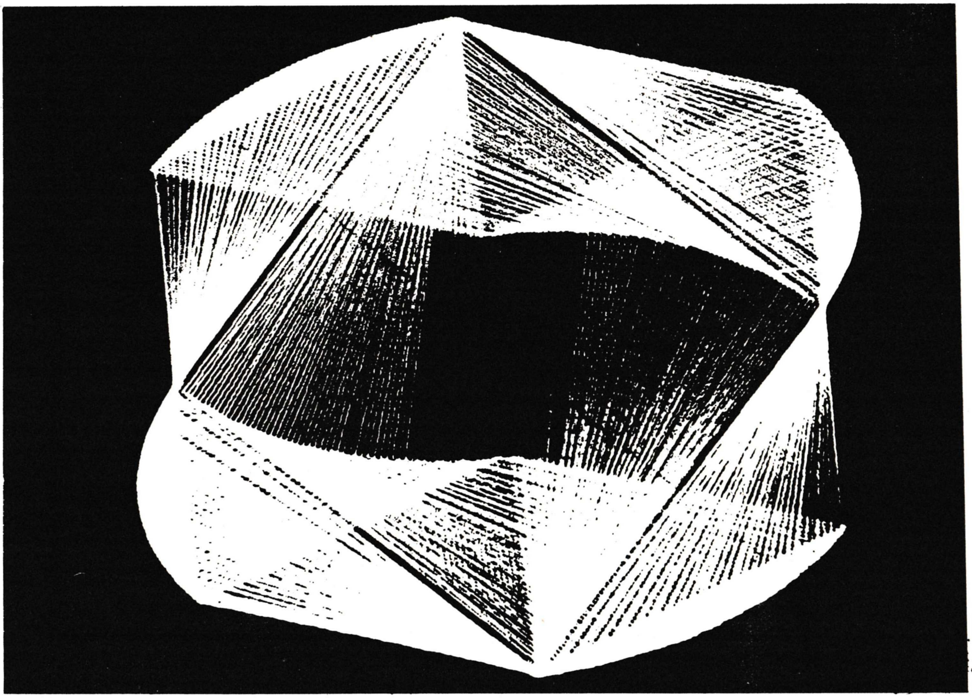
Rotating a Cube by Computer

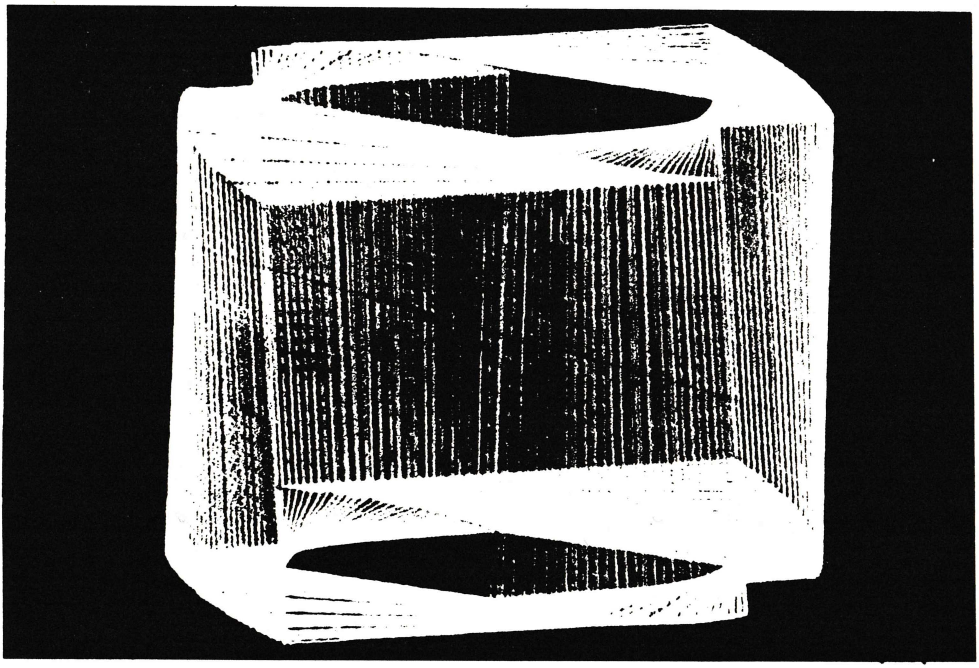
by B. M. Thompson and A. G. McKernan

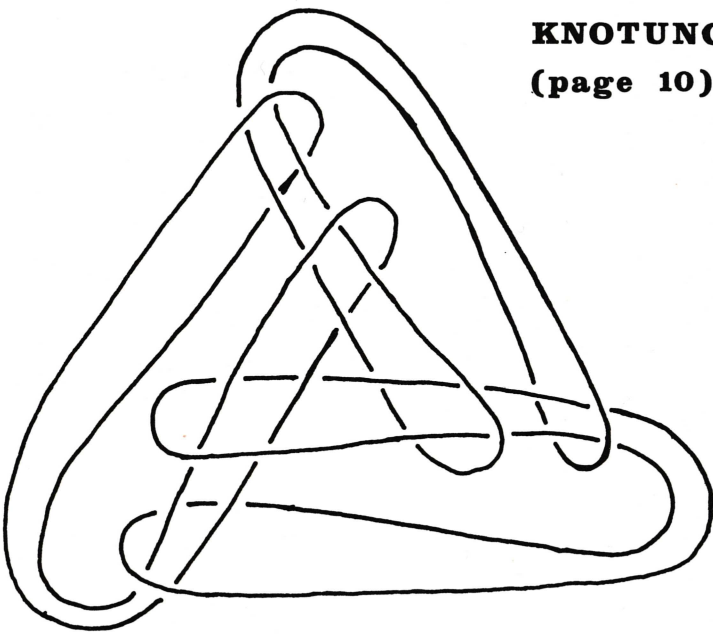
The 3D Program was written to enable a rotating polyhedron to be displayed on the screen of the PDP7 computer. The polyhedron is specified by a description of all the lines which constitute it, each line being specified by the six cartesian co-ordinates of its two endpoints. Every 0.1 second the program examines the controls of the computer to find out the operator's choice of speed and axis of rotation (which can vary with time). For the appropriate number of times it then applies to the specification of the polyhedrion each of three transformations, corresponding to rotations of 0.01 radians about each of the three axes. The program then translates the new specification into a program to display a projection of the polyhedron along one axis, and makes the computer's display unit obey this program.

On the PDP7 at the Mathematical Laboratory the display unit, thus set, operates independently of the main program, and so at this stage the program waits for the next 0.1 seconds and then starts again. The three pictures we show were produced by photographing the screen with an exposure lasting for a complete revolution of a cube, and showing each step clearly. The cube was rotated about its (1, 1, 1), (1, 0, 3), and (1, 2, 3) axes in the upper and lower photographs opposite and that on the cover respectively.

We are deeply grateful to Neil Wiseman of the Mathematical Laboratory, without whose help and encouragement none of this work would have been possible.







KNOTUNG
(page 10)

Der Ring der Algebrungen

by A. G. Smith

PART IA

The three fluid dynamicists, Friedlinde, Badtschella and Bredertung guard the magic Goldstein in an inviscid, irrotational fluid.

The evil Moffadt, forswearing rigour, seizes the Goldstein, and with its aid, and the prophecies of Taunt, constructs a magic Ring.

The giants Damtp and Edsach, builders of Hodschhalla, seize the TMS apples, and carry them off to Titanheim.

The Department travels west to Moffadthöhle, and tricks Moffadt into changing into a self-gravitating mass. When he collapses, he is easily captured, and Hodsch seizes the ring and Goldstein.

The Department gives Damtp and Edsach the ring and Goldstein, and recovers the TMS apples. The giants quarrel, and Edsach is closed down. Damtp changes into a heraldic beast: the unicorn Konvergenz.

The Department returns to Hodschhalla viâ a short-cut through Artzschule.

PART IB

So heroic are the deeds of Siegton, that he rises to become President of TMS.

Siegton falls in love with Deia, who is engaged to Leverrier, leader of the Adamungen. The rivals meet in battle on the Cricket Field.

Hodschvater has been persuaded by the Adamungen to intervene on Leverrier's behalf, and sends the Research Student Grünhilde to carry out this task.

Grünhilde, however, is persuaded to join TMS as an Associate Member, and the Adamungen are scattered.

The Department banishes Grünhilde to a Northern fastness watched over by Kartrecht, and Siegton is condemned to work in Mathlab.

PART II

Siegheil, offspring of Siegton and Deia, is studying under Licquorice, whose ideal it is to knot the ring.

While Licquorice is calculating the group of the Gordian knot, Hodschvater appears, and interprets the strange runes Kassels had written; he tells Licquorice that only one who does not understand fudging can slay the Unicorn Konvergenz and gain the ring.

Siegheil's Polkinghorn-call is heard from afar as Licquorice examines his supervision-work. He finds that Siegheil has never fudged an example.

Siegheil finds the magic one-sided sword Roseblade, and is directed to Konvergenz's lair. The monster reads the minutes of the preceding two sections, which Siegheil signs as a true and accurate account. Siegheil then slays the unicorn, gaining the ring and Goldstein.

Licquorice tries to knot the ring, and Siegheil kills him.

Ladyj invites Siegheil to a tea-party, where he meets Grünhilde.

PART III

Three uniformly equivalent Norms, the daughters of Bardung, foretell the fall of Hodschhalla.

Siegheil is cycling South in search of adventure. As he passes the domed Hall of the Neu-ungs, he is hailed by the hero Fitzwillung. Siegheil is given a draught of the ill-omened fluid known to the Quintics as coffee, which causes him to forget Grünhilde and fall in love with Fitzwillung's half-sister Girtrune, the daughter of Moffadt.

The beautiful Girtrune lures Siegheil to a tea-meeting in the Court of the Neu-ungs. Siegheil is cast into a deep sleep by the speaker, and then run through with Girtrune's toasting-fork.

Grünhilde takes control, and has Newton's Apple-Tree hewn down to build Siegheil's funeral pyre. She sends two of her supervisees to Hodschhalla with their gowns aflame.

Grünhilde rides into the fire on Siegheil's bicycle. An inviscid, irrotational fountain that is to hand falls down and overflows, and the fluid dynamicists emerge to recover the Goldstein.

As Hodschhalla burns down, the staff are seen serenely drinking Departmental Tea, seated in the Common Room.

Incidence Incidents

by H. T. Croft

This is a collection of (other people's) bright remarks, elegant theorems, and plausible conjectures on some 'incidental' aspects of intuitive geometry that deserve to be more widely known.

In the Educational Times of 1893, Sylvester proposed the following problem: In a real Euclidian plane a set of n points is such that the line joining any pair of the set always contains at least one more point of the set. Prove that the n points are collinear.

The problem was not solved satisfactorily for 50 years; several proofs have since appeared. The two reductio ad adsurdum proofs given here are due to Grunwald and L. M. Kelly respectively. See (2)

First Proof. Suppose such a non-collinear configuration did exist; embed it in the real projective plane; project one of the points to infinity; then the lines joining it to the others are now parallel; each of these parallels contains at least two points of the set, by the given hypothesis. Consider that joining line that makes the least (non-zero) angle α with this set of parallels; it contains, by hypothesis, at least 3 points of the set, each on one of the parallels, say A, B, C in this order—see figure 1. Let B$'$ be the other (or another) point of the set on the parallel through B; then according as to which side of B B$'$ falls either AB$'$ or B$'$C makes an angle strictly less than α with the parallels. This contradicts the minimality of α.

Second Proof. Consider the set of (non-zero) perpendicular distances from points of the set to the joining line. Let the least such distance (or one such) be that from A to the line through B, C, D, points of the set (in this order). Without loss of generality, say C & D lie the same side of the foot P of the perpendicular from A—see figure 2. Then a little elementary geometry shows that the perpendicular distance of C from AD is strictly less than AP. This contradicts the minimality in the definitions of A and P.

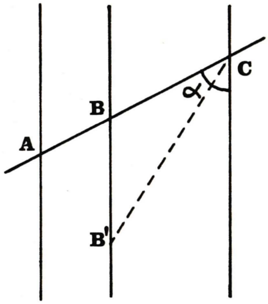


figure 1

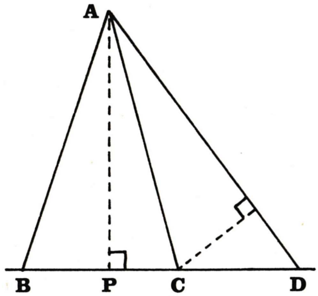


figure 2

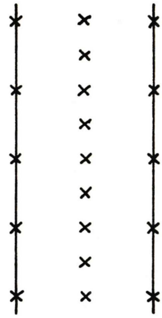


figure 3

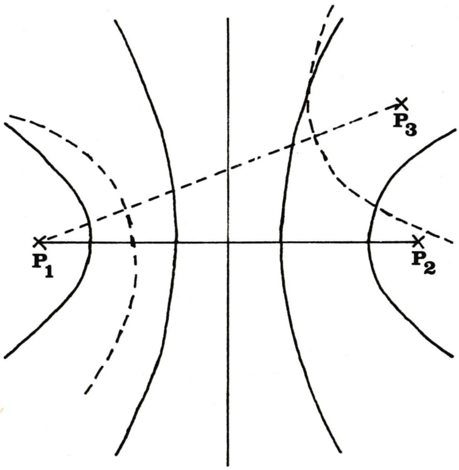


figure 5

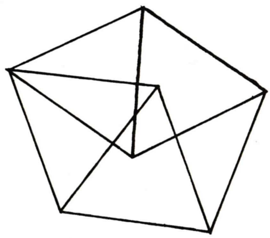


figure 6

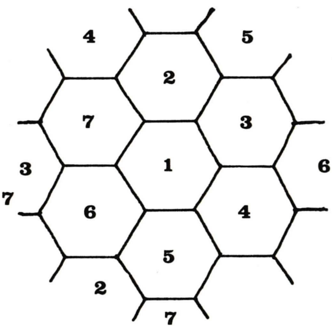


figure 7

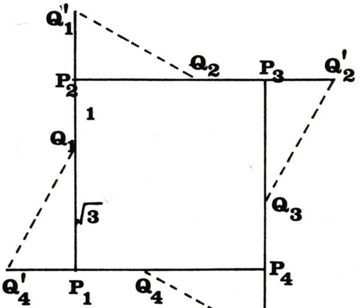


figure 4

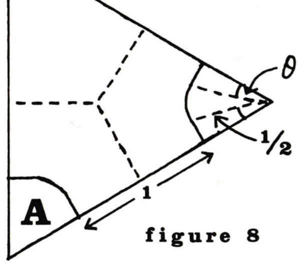


figure 8

Remarks: (1) Logical status of the problem

Now we observe that in the (projective) complex plane, Sylvester's Theorem is false: indeed the 9 points with homogeneous co-ordinates given by $(0, -1, \omega)$, $(0, -1, \omega^2)$, $(0, -1, 1)$, $(\omega, 0, -1)$, $(\omega^2, 0, -1)$, $(1, 0, -1)$, $(-1, \omega, 0)$, $(-1, \omega^2, 0)$, $(-1, 1, 0)$, where ω is a complex cube root of unity, lie 3 by 3 on 12 straight lines and thus provide a non-collinear Sylvester configuration. Considering this fact in another light, we see now why no 'obvious' proof of the impossibility of a Sylvester configuration in the real plane is available: we have to use axioms not available in the complex case, in particular those of order. Proofs depending explicitly on the axioms of order and avoiding the projective or Euclidian ideas of the above have been given. (As Professor Littlewood has pointed out, another notoriously difficult theorem, that 'if two angle-bisectors in a triangle, drawn as far as the opposite sides, are equal, then the triangle is isosceles' is true only if we restrict ourselves to real co-ordinates. Sylvester opined (wrongly) that no direct proof of this could exist; those known are long, needing the axioms of order.)

(2) Variants on the theme

(a) If 'finite' is replaced by 'countably infinite', then trivially (2), (3) non-collinear Sylvester configurations exist—e.g. the vertices of an infinite square lattice. The same is true even if we restrict the points to lie in a strip of the plane—see figure 3.

(b) There is a dual form: if a finite system of lines in the real plane is such that through any point of concurrency of two lines passes a third, then the lines all concur. For proof, we may reciprocate in any fixed circle (or conic), and arrive at the original problem. Or we may proceed directly, rather as in the second proof above: consider the nearest intersection to any particular line of the set. See (3).

(c) We may deduce a theorem on circles by inverting in a general point in the plane. But more interesting is: if a finite set of circles in the real plane is such that through any pair of intersection-points of any two circles of the set there passes a third, is the set necessarily a coaxal family through two common points? (Trivially no, if we allow a countably infinite set: all those circles with equations $f(x, y) = 0$ with all the coefficients rational; through the points of intersection of $f = 0$ and $g = 0$ pass also $f + \alpha g = 0$, α rational, all being circles of the set). But rather surprisingly the answer is no in the finite case also. Take the 8 vertices of a cube; they lie 4 by 4 on 12 circles (in space) which themselves lie on the cube's circumsphere; now invert in a general point of this circumsphere: the resulting coplanar configuration of 12 circles, forming, 3 by 3, 12 coaxal sets, provides the desired configuration. This remark is due to Segre.

(d) If a bounded closed set of points is such that the axis of symmetry for any two of its points is always an axis of symmetry for the entire set, then all the points lie on a single circle. Hint for proof: take the smallest circle that covers the set, and show that no point of the set can lie in its interior, for that would imply some other point lying outside. For details, see (4); this book is a fascinating introduction to many branches of intuitive geometry.

(e) Some years ago I asked: can a finite set of n points in a plane be such that the perpendicular bisector of every pair of them passes through at least 2 other points of the set? Professor Kelly (6) has suggested the elegant configuration of the points $Q_1, Q_2, Q_3, Q_4, Q_1', Q_2', Q_3', Q_4'$, of figure 4.

Problems of this type have much in common with many problems of elementary number theory, not only in their trivial-or-impossibly-difficult character, but also in that although it may be difficult to find one odd solution as above, it is a problem of a different order of magnitude whether such are 'flukes' (for certain small n), there being no solutions for sufficiently large n, or whether they are the tip of an infinite iceberg of solutions, as yet hidden from us.

Another class of problems concerns rational or integral distances between points in the Euclidian plane. A notorious unsolved problem of Ulam is: does there exist a set of points dense in the plane with the distance between any pair of them rational?

Problem 12 (page 23 of last year's EUREKA) asked for (i) configurations of n non-collinear points, all integral distances apart, for any given n; and (ii) a proof of the impossibility of such a configuration for an infinite number of points.

PROOF (i) Take the n points on a circle radius R, thus: $P_k = (R \cos k\delta, R \sin k\delta)$, where k runs from 0 to $n-1$, $R = (n^2+1)^n$, and $\delta/2$ is the acute angle whose sine is $2n/(n^2+1)$, and cosine $(n^2-1)/(n^2+1)$. Then the points are distinct, for it is not hard to see that $(n-1)\delta < 2\pi$, and trivially are non-collinear. And $P_kP_{k'} =$ $2R \sin \tfrac{1}{2}|k-k'|\delta$, which is 2R (a polynomial in $\sin \delta/2$ and $\cos \delta/2$ of degree at most $|k-k'| < n$), and this is integral.

(ii) (see (5)) Suppose that there were an infinite, non-collinear, integrally separated set of points P_i in the plane. Select some non-collinear triplet, say P_1, P_2, P_3. Now, the points P_i $(i > 3)$ all lie on a set of hyperbolae (some possibly degenerate), foci P_1, P_2, for $P_iP_1 - P_iP_2$ is a (positive or negative) integer c_i; and moreover there are only a finite number of such hyperbolae, for $|c_i| < P_1P_2$, by the triangle inequality— see figure 5. Similarly P_i $(i > 3)$ all lie on a finite set of hyperbolae with foci P_1, P_3. So P_i is a finite set, contradicting the hypothesis, since any hyperbola of one set intersects any one of the other in at most four points (their foci are not collinear).

A long-standing open problem of Erdös is whether the plane can be split into four disjoint sets such that no two points from the same set are ever unit distance apart. L. Moser has observed that this is certainly impossible with '4' replaced by '3': for, in figure 6, in which all the lines have unit length, if the vertices were coloured with but three colours, then some two adjacent vertices would be isochromatic. On the other hand, the dissection is possible with '4' replaced by '7': take the infinite regular hexagonal lattice, of side x (any fixed x satisfying $1/\sqrt{7} < x < \tfrac{1}{2}$); colour one block of 7 hexagons (one central, and its neighbours), then translate this colour scheme to colour all such blocks; one verifies easily that this satisfies the conditions. See figure 7.

Moser has varied the question: how 'dense' can one plane set that realizes no unit distance be? (Upper) density is defined, say, as the (upper) limit of the measure (area) of that part of the (plane measurable) set that is contained in a very large circle divided by the area of the circle. By translating (without rotation) Moser's framework (figure 6) about the interior of the large circle, and observing that edge effects are ne negligible, we obtain that the density δ of any such set satisfies $\delta \leqslant \tfrac{2}{7} \doteq .2857$. I exhibit a configuration with density .2293, which shows that $\delta \geqslant .2293$ (the bounds are surprisingly close).

Let the set consist of (topologically open) convex lumps around the vertices of the infinite equilateral triangular lattice, the lumps being congruent, each the intersection of a circle and a regular hexagon, both centered on the vertex. See figure 8. Clearly the density = area A/($\tfrac{1}{3}$ area triangle). This equals

$$\frac{\sqrt{3}\,(\pi/6 - \theta + \tfrac{1}{2}\sin 2\theta)}{(1+\cos\theta)^2}$$

where θ is as shown. The set is seen to satisfy the given conditions; θ is at our disposal. On differentiation, we find that the maximum density occurs at $\theta = \theta_0$, where $\theta_0 + \sin\theta_0 = \pi/6$. The density is then, on substituting back, seen to be $\sqrt{3}\tan(\theta_0/2)$. With the use of tables, we find $\theta_0 \doteq 15°5'$, density $\doteq .2293$, as stated. For $\theta = \theta_0$, we find that the boundaries of the 'lumps' have their straight parts and curved parts of equal total length, apparently a quaint quirk.

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Book Reviews —continued from page 40

INTERNATIONAL TABLES OF SELECTED CONSTANTS. No. 15—DATA RELATIVE TO SESQUITERPENOIDS. By G. Ourisson, S. Munavilli, and C. Ehret. Preface by F. Sorm. (Pergamon). 90s.

This excellently produced book is an indispensable reference volume for all those working in the as yet relatively unexplored field of sesquiterpenoids. The wealth of useful information on sub-groups, based on over 40 different basic structures, is so well-presented as to make this book a joy to read. An excellent index and bibliography are provided. However, at 90s., the book is possibly a little beyond the average research student.

D. G. COOPER

VECTOR ANALYSIS. By B. Spain. (Van Nostrand)

This is intended as an introductory text to the subject of vectors. From the point of view of the reviewer (namely that of a physicist) the text was illuminating, rigorous enough to be useful (compared with the very elementary approach), yet not so rigorous as to be stultifying. The subject matter dealt with is what one would expect from a book of this nature (grad, div, curl; integral theorems; applications to mechanics; &c.), and on the whole the methods used kept sufficiently in touch with the physical applications to remain intelligible. The treatment was surprisingly lucid and, above all, readable.

J. R. MASON

SYSTEMS OF LINEAR EQUATIONS. By B. E. Margulis

This is a very useful guide to ideas about, and solutions of, systems of linear equations. There are plenty of examples and the mathematics needed to understand every section is learned at school. Even so, the subject is treated fully and there are several interesting sidelines and exercises for those who are so minded. Subjects treated include successive elimination, determinants, approximate solutions, inconsistency and graphical methods.

P. D. COOPER

HIGHER CALCULUS. By F. Bowman and F. A. Gerrard. (Cambridge U.P.) 60s.

This book seems to approach analysis from the applied mathematician's viewpoint—in fact, it is much better to regard it as a text on mathematical methods rather than on analysis, for which it is not rigorous enough. As such it is a solid, thorough treatment of the parts of calculus (except differential equations) covered in, and required for, first year mathematics.

A. KALETZKY

Leave it to Aunt Minnie

a fantasy by I. N. Stewart

As usual, the Stellar Survey Service had a problem. Out of about 4 billion planets surveyed, 3.7 billion had presented it with problems. And this one was no exception.

The gravity was near Earth-normal. The atmosphere was breathable. The inhabitants were friendly. No, it was nothing so simple as that. It was the strange behaviour of the inhabitants that was the problem.

They were small cubes, about a foot across. This wasn't the problem either. 0.1 per cent of known intelligent creatures were geometrically regular in form. Nor was it their continual motion over the surface of the planet. But this must be described in more detail.

Each cube appeared to have a definite path over the planet, along which it moved at a speed of about 12 feet per minute. When one of them was picked up bodily and moved off this path, it promptly exploded (and didn't seem too pleased, either!). Certain of the cubes, mainly those in the mountains, followed the same path over and over again, retracing a closed curve round the mountainside. One or two, perched on the very tops of the mountains, didn't move at all. The same thing happened in depressions, some going round and round the slopes, and others staying at the bottom.

The repetition was not exact. Occasionally, the cube would be higher up or lower down than usual. The only thing this seemed to fit in with was somewhat unlikely. When the planet's moon was overhead, the cubes would be lower down the mountains than when it was not. This behaviour appeared to be regular, and there was no other local phenomenon with a similar correlation. So there was a sort of tidal effect, but in the opposite direction to normal tides.

Even this was not the main problem, which was: Why, as soon as the Survey ship landed, did the nearest cube greet the emerging Earthmen in English? How had it learned the language? No, I hate to disappoint you, but the cubes were not telepathic. And that was the problem. How does a non-telepathic race greet you in your own language, without having heard it before?

And, as in the other 3.7 billion problems, the SSS did the only thing it could do. Hand the problem over to AMAB.

Frad Pzz't-Vlu was seated at his desk in one of AMAB's information centres, studying the thin tape that wound its way out of the Subwave Communicator Complex.

'P/QAG7795164—CAT URG—PRIORITY 6. SCS REPORT STAR R25/α SCORP NOVAED: POSTMORTEM TELEPATHY INDICATES INHABITANTS INTEND TO SUE SCS AS SOON AS REVITALISED SINCE STAR WAS UNDER GUARANTEE FOR 10^7 YEARS, UNEXPIRED. SCS REQUEST SOLUTION UTMOST SPEED END.'

The Stellar Control Service, eh? At 10 per cent, that would be a nice fat cheque. And the answer was obvious. Use a timejumper to go back to before the star novaed, find out why the control satellite failed, and repair it. But maybe that was too easy. He punched the console keys in the pattern that would initiate a search through the files for the SCS:R25/α SCORP contract.

AMAB was a galactic-wide organisation devoted to problem-solving. The initials were those of 'Aunt Minnie's Advice Bureau', a relic of its origins amid the columns of a cheap women's magazine. It would take too long to go into the details of its meteoric rise to prominence. Now it took on problems involving entire galaxies without a qualm. But it still had time for less important or extensive problems, like the one coming out of the Subwave now:

'P/QAG7795165—CAT TRIV—PRIORITY ZERO. PIONEERS ON 45Γ'p UR MIN/3 REPORT PLANET INHABITED BY TWO SENTIENT TIDAL WAVES—ONE MALE, ONE FEMALE. REQUEST SUGGESTIONS FOR NAMES END.'

Frad left that one to the miniputer, but chuckled when he saw the answer' 'CALL THEM EBENEEZER AND FLORENCE—EB AND FLO FOR SHORT.'

The Subwave chattered again. It was the SSS's report on the planet with cubical inhabitants. Frad loosened his monomolecular necktie, and set to work. Three hours later, surrounded by a rapidly-growing pile of paper, he thought he had the answer. So he sent off a message: 'AMAB TO SSS—DO CUBES HAVE PRECOGNITIVE POWERS?'

'GOOD TRY, BUT NO, ' came the answer. 'ESPERBOX SHOWS ONLY USUAL LOW TRACES—NO DEVELOPED PSI-FACULTY OF ANY KIND.'

He was not greatly pleased, therefore, when the contract between the SCS and R25/α SCORP arrived, and he discovered that it explicitly forebade use of timejumpers in the system for any purpose whatever. It appeared that the inhabitants were Antichronists—a weird religious sect which believed that timejumping was meddling with the plan of the Universe.

A straight recon job on the planetary system would cost SCS 10^{13} megacredits at least, while damages in the forthcoming action would be at most 10^{12} Mc. SCS could afford it, but unless they kept it out of court their public image would suffer. There was always their competitor, Planetary Maintenance Ltd., to think about. Frad referred the problem to the Legal Loophole Dept., but he wasn't very hopeful.

By the end of the day, he was practically buried beneath a mound of paper—piles of it, screwed up balls of it, 'putertapes of it, wpb's full of it. Two miniputers had gone psychotic, another had exploded, and the main computer had had to be forcibly restrained from committing suicide by dividing 1 by zero aleph-null times in succession, channelling the output back to the input, and starting all over again. It was now being thoroughly overhauled. Frad, together with four laboratories' full of the cream of AMAB's technical staff were in much the same state, and no nearer a solution. The first inhabitant of R25/α SCORP had been revitalised, and was threatening to tell the newscasters. And the Legal Loophole Dept. had no ideas on their problem either.

So round to Joe's bar he went.

'Hi, Joe!'

'Lo, Frad! The usual?'

'Please, Joe.'

Joe rapidly mixed half a litre of Venusian Swampjuice with a thimbleful of Grootwater, two drops of Fffthffl, and a slice of synthelemon. He heated it for a few seconds in an infrared laser-beam, quenched it in liquid argon, and poured it out into a square saucer containing two chips of orange ice and a cocktail cherry with two sticks in it. 'Here we are Frad, one Snurjit's Downfall. One and three fifths, please.'

'Thanks, Joe. Say, Joe, any good at problems?'

'You mean, like: 'What's grey, got 11 legs, and moves faster than light? 2.75 elephants in a matter-transmitter.' That sort?'

'Not quite, Joe. More like this.' And he poured out his troubles to Joe's sympathetic ear.

'Ah,' said Joe. 'You'll want the Prof. I'll get him.'

A few minutes later he returned with a bespectacled, slightly wild-looking man.

'May I sit here?'

'By all meansh,' replied Frad, now well and truly inebriated. 'Make yourshelf at home.'

'Thank you. My name is Dumble. Aloysius Dumble. They sometimes call me "Prof".'

'You an academic, or shomething?'

'No, no. I'm a practical man. An inventor. Unfortunately, none of my inventions have worked yet. But I'm sure the latest one will. It's an Electronic Intuitive.'

'A what?'

'Electronic Intuitive. It builds up a complete picture from remarkably few facts. For instance, it could work out a person's state of health from a glance at his shoelaces. The trouble is, I can't find a problem worth testing.'

'Lishen to mine, then,' said Frad. 'And have a jrink.'

And he told the Prof all about it, punctuated frequently by liquid refreshment. Finally the Prof persuaded him to come and see the machine, and try it out. Frad was by then drunk enough to try anything.

So, arm in arm, they staggered out of Joe's, along the street, into an apartment-block, up 17 flights in a service lift (they couldn't find the right one), and into the Prof's room. There, in the middle of the floor, was the most complicated mass of wiring that Frad had ever seen. 'Thash it,' said the Prof. 'Try it out.' So Frad did. It clicked and squeaked, lights flashed off and on, and finally it came out with:

≪ Try time jumper theory≫.

'Oh, that's no use!' cried Frad. 'The contract won't let us use a timejumper!' He had forgotten by now that he had asked about the cubes, and though it was referring to the other problem, the R25/α SCORP contract.

Next morning, back at work, Frad suddenly realised that the Prof's Intuitive had been answering the cube question. But how could a timejumper help? He thought hard. Timejumper <u>Theory</u>, it had said. Well, anything was worth a try. He asked for information.

Apparently the basic idea had been worked out some time back, by a fellow called Einstein. Relativity. He was wrong, of course, but his ideas were good enough to suggest the idea of a timejumper to whoever had invented it. The idea, it seemed, was that time was just another dimension. It was subjectively different, in that in some sense humans moved along it at a uniform rate, but were free to move at will along the space-dimensions. The timejumper merely moved at will along the time-dimension— or at any rate, a close approximation to it.

Now, he thought, suppose the cubes are different. Suppose they are constrained to move in space, but free to move in time. Then they would know English, by going forward in time to when they were taught it, and returning with it remembered. Their strange motion would be the space-constraint, of course. He fed the equations from the timejumper manual into the miniputer, to see. But the computer seemed to think they would move in a straight line, right off the planet. Then he realised that, firstly, relativity wasn't correct, and secondly, he had neglected the planet's gravitation. He handed the problem over to the technical staff. Two hours later, he had the answer. On the most modern theory, the cubes would have to move on equipotentials in the gravitational field, on the surface of the planet. Hence the motion in closed paths, the effect of mountains and hollows, the immobile cubes at gravitational maxima or minima. The tidal effect was indeed precisely that—the moon's motion modified the equipotentials. And the explosions when cubes were moved was a manifestation of the energy differential between different potential levels. It all fitted. Now to test.

He sent a message: 'AMAB TO SSS. CHECK CUBES FOR TEMPORAL TRANSITION ACTIVITY:'

Back came the reply: 'ACTIVITY HIGH. HOW?'

So he told them his theory. A quick check established that the paths were indeed equipotentials. One problem dealt with. Perhaps the Electronic Intuitive could solve the other!

Frad rushed over to Joe's. The Prof met him at the doorway. 'It worked,' said Frad.

'Eh? What worked?'

'The Intuitive. Can I use it again?'

'Of course. Yes. No.'

'No?'

'I took it to bits. I didn't realise it was working.'

'Pity. Reassemble it, anyway. AMAB wants to buy the plans. See you!'

And back to work he went.

Several million inhabitants of R25/α SCORP had now returned to the land of the living, and were clamouring for blood. Only quick action by the Galapol in reminding them that nothing could be done until all were alive again prevented a disaster. But soon they would all be alive, and the trouble would start, and AMAB would lose its ten per cent, which was—by the cringe—10^{11} megacredits!

Was there some other way of travelling in time, other than a timejumper? Well, there was Schmidtski's Temporator, or a Chronocar, but they wouldn't work under the radiation conditions prevailing. And no other means of time-travel was known.

Except...

... for the cubes!

'AMAB TO SSS. PUT ME IN TOUCH WITH A CUBE.'

He took the call on direct visual link.

'I've got a proposition for you,' said Frad. He told the cube the problem. 'What we want you to do is to go back in time, and put the control satelite right. In return, we provide you with a means of moving in space. What do you say?'

'Don't need it,' said the cube, turning a somersault. 'I went forward in time to when you provided me with it, after I'd done the job, so now I don't need your help.'

'All right,' said Frad. 'In that case, I won't tell you how.'

The cube promptly began moving along an equipotential.

'That's not fair,' it said.

'Then do the job.'

'And then you'll tell me?'

'Yes.'

'OK then. You won't need to. I've remembered again. But for Orgzd's sake don't change your mind again and leave me stranded somewhere in space!' And the cube disappeared.

Frad relaxed. The Galapol reported the sudden disappearance of several million inhabitants of R25/α SCORP from Revit Centre. SCS presented AMAB with a cheque for 10^{11} megacredits. Small cubes began appearing in the unlikeliest places. Everything was wonderful.

The Subwave Communicator Complex burst into life.

'P/QAH0000237—CAT UNORTHODOX—PRIORITY 3. SSS REPORT PLANET UNDER SURVEY CHANGES SIZE FROM 3000 KM. TO 6000 KM. DIAMETER EVERY 3.68 DAYS, EXCEPT ON BANK-HOLIDAYS AND BIRTHDAYS OF CREW-MEMBERS.'

'Joe,' said Frad...

The Paradox of the Surprise Examination

by D. R. Woodall

Many paradoxes, such as the traditional 'proofs' that all triangles are isosceles or that $0 = 1$, have become so familiar to mathematicians that they have long ceased to puzzle. Others, such as Russell's paradox or the statement 'I am lying', do not seem capable of complete resolution without the introduction of new rules and conventions into the language. The paradox I wish to consider here seems to me to be especially interesting in that the fallacy is sufficiently obscure for it still to be puzzling to most people, but it is nonetheless susceptible of resolution by straightforward logical argument. The paradox is the following.

> A school-teacher informs his class on the first day of term that they will be set a surprise examination on one day during the term, and he explains that he means by this that they will not know, when they come to school on the morning of the examination, that the examination is to be held on that day. The children reason inductively as follows. 'The examination cannot be held on the last day of term: for if we were to come to school on the last day without already having had the examination, we would know that it must occur on that day, and it would not be a surprise. Equally, it cannot be held on the second last day: for if we were to come to school on the second last day without already having had the examination, and knowing that it cannot be held on the last day, then we would know that it must occur on the second last day, and so it would not be a surprise. We can show by a repetition of this argument that the examination cannot be held on any day during the term.' In practice, of course, what happens is that teacher sets the examination on the twenty-third day of term, and the children are surprised!

I would suggest that anyone who has not previously met this paradox, or one of its variants, should consider it for a few days before reading any further.

The wording of the paradox is actually slightly misleading, as the following argument shows. It goes without saying (at least, it has up to this point) that the children must assume that teacher is telling the truth. However, their argument clearly shows that he is lying, as it shows that there cannot be an examination during the term, while he asserts that there will be. This is a contradiction. Thus it is important to realise that the true paradox does not lie in the conflict between theory and practice, which is emphasized by the above wording, but in the fact that the theory by itself is self-contradictory. Thus either one of the children's hypotheses is false, or their reasoning is faulty. (This last assertion of course requires an act of faith).

In fact, if we examine the children's argument more closely, we see that they have made an even stronger assumption than that mentioned above. The hypothesis that teacher is telling the truth is not sufficient for the children's argument to follow through; for it is not absolutely impossible, on the basis of this hypothesis alone, that the examination could be held on the last day of term. The children must make the hypothesis that they know that teacher is telling the truth, in fact that they are absolutely certain, without any possible doubt whatever, that he is telling the truth in every minutest detail. It seems to me that, if this hypothesis is made, the children's argument is logically correct, and leads to the contradiction mentioned. Since the logic is correct, this simply means that the hypotheses are inconsistent. That is, the two statements

1. Teacher says that there will be a surprise examination this term (in the sense explained above); and

2. The children are absolutely certain that teacher is telling the truth;

are mutually inconsistent. Note that the two statements

1. Teacher says that there will be a surprise examination this term; and

2. Teacher is telling the truth;

are not mutually inconsistent, and may indeed both be true in practice. The children's argument does not enable them to deduce that, if teacher makes the reported statement, then he is lying. What they can deduce, and indeed have proved quite rigorously, is that, if teacher makes the reported statement, then they cannot possibly know with absolute certainty that he is telling the truth; and this is the situation that arises in practice.

I must in all fairness add that, while the logic of the above explanation seems to me to be inescapable, it has by no means been found to convince everyone to whom I have propounded it!

Problems Drive Answers

(A) (1) $u_n = (n-1)(n-2)(n-3)(n-4)$ (2) $u_n = n\,(u_{n-1}) + (-1)^n$
(3) $u_n = k$, where $k-1, k+1$ are prime
(4) 24, 30, 120, 240, 1200, 2400 (monetary units currently being issued)

(B) (1) 3 (2) 778

(C) Neither; A Damtp, B Dpmm, C Stat (no road can lead through an inpenetrable jungle !)

(D) (1) Infinity (2) e.g. 'Knotung' (p20) and figs. 1 & 2

(F) $f_n(x) = \tan(\tan^{-1} x + \pi/n)$ $(x \neq \infty)$

$f_n(\infty) = \tan(\pi/2 + \pi/n)$

where $-\pi/2 < \tan^{-1} x \leqslant \pi/2$, $\tan \pi/2 = \infty$

(G) See figure 3 $(n \neq 1)$, figure 4 $(n = 1)$

(H) $e^{-1/25}$

(I) e.g. figure 5

(J) 1, 2, 4, 8, 16, 31 (sic)

(K) e.g. figure 6

The winners were B. Scorer and A. Manning, with 61% of possible marks.

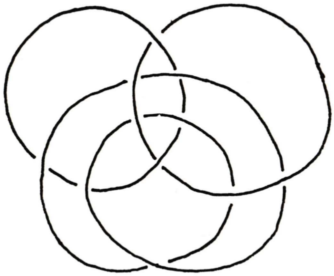

figure 1

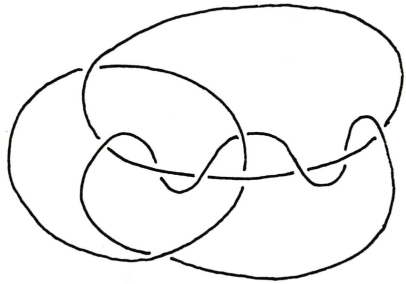

figure 2

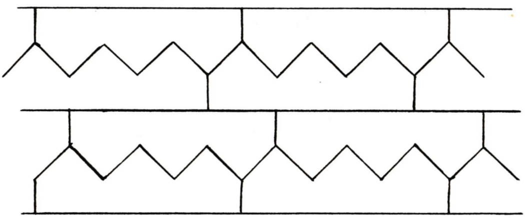

figure 3

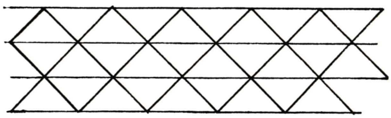

figure 4

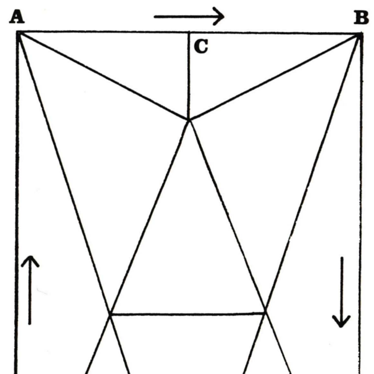

figure 5

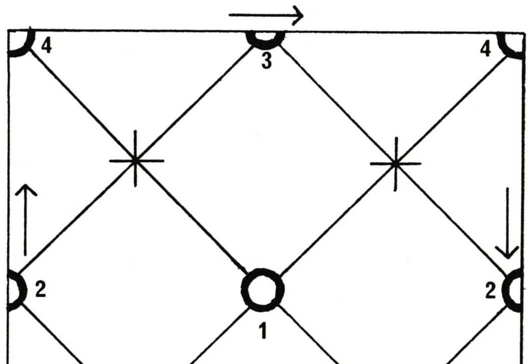

figure 6

The Archimedeans

The Archimedeans have had another successful year, with very good attendance at evening meetings. The talk by Professor Ryle on 'Quasars and Radio Galaxies', the Careers Meeting organised by Mr. J. N. Coope and held as a joint meeting with the Overseas Information Group, and Professor R. S. Scorer's talk on 'The Mechanics of Hurricanes' were particularly notable. Amongst the Tea Meetings, special mention should be made of Dr. Bretherton's talk, given at short notice to replace Dr. Taunt who was prevented by illness from giving his talk on 'Straight and Crooked Thinking', and the talk on 'A Non-Archimedean Universe' given by Mr. J. T. Knight, which confirmed our worst suspicions about valuations.

The Problems Drive was, as last year, a great success, with a number of 'Invariants' from Oxford competing. The Computer Group has kept up the attendance at its weekly meetings, and has been very busy programming for Titan. The Puzzles and Games ring has been revived; it has thought up many puzzles and solved some of them. The Bridge Group is still holding regular meetings, but the Music and Chamber Music Groups have lapsed. It is hoped to revive the Music Group next year, and to form a Literary Group. The Tiddleywinks Match against the Dampers, and the Punt Party were both very successful, but again some visits had to be cancelled for lack of support, and those that were not were rather poorly attended. The Bookshop has continued to provide a service to undergraduate mathematicians, and has had a good year.

This years evening meetings start with Dr. Bryan Thwaites, The Principal of Westfield College, London, who will talk on 'Solve: $x - y = 1, x - 2y = 0, 2x - y = 3$'. This meeting will be on Wednesday, 11th October, not a Friday as usual. We also hope to have a Careers Meeting on the lines of those held in previous years, and arrange a visit to Oxford during the Michaelmas Term. The Problems Drive will again be held in February; the 'Invariants' will be challenged to take part in it. A Tiddleywinks Match against the Dampers and a Punt Party to Granchester will be arranged as usual. Visits are being arranged for next year, and if there is enough support there may be trips to some London Theatres.

It is hoped that all members of the society will find something in the coming year's programme, but all suggestions as to possible changes in future years will be welcomed. These should be made either directly to the secretary, or through the suggestions book kept in the Arts School.

A. C. NORMAN, Secretary.

Mathematical Association

22, Bloomsbury Square, London, W. C. 1.

President: A. P. Rollett, M.Sc., F.I.M.A.

The Mathematical Association, which was founded in 1871 as the Association for the Improvement of Geometrical Teaching, aims not only at the promotion of its original object but at bringing within its purview all branches of elementary mathematics. The subscription is 2 gns. per annum; for students and those who have recently completed their training junior membership is available at 10s. 6d.

The Mathematical Gazette is the journal of the Association. Published four times a year, it deals with mathematical topics of general interest. The present Editor is Dr. E. A. Maxwell.

Book Reviews

THE AUTOBIOGRAPHY OF BERTRAND RUSSELL, 1872-1914. (Allen & Unwin) 42s.

This book is disappointing. Here is a man who has combined three brilliant careers into one lifetime: first as mathematician, then as philosopher, and finally, with less success, as politician. In the first 42 years of his life, he and a few others virtually rewrote the whole of mathematical logic and started off many trains of thought which are still being developed today. Yet all he gives as an account is a rather disconnected series of anecdotes, some amusing, many boring, many in downright bad taste, concerning the sexual and other idiosyncrasies of his contemporaries.

Nearly half the book consists of rather boring letters. The most important letter Russell ever wrote ('Dear Frege. Consider the set of all sets which are not members of themselves...') is not included; indeed, very little correspondence illustrating his mathematical development is there. This autobiography may not be primarily for mathematicians, but is it for anyone but the scandalmonger? C. J. MYERSCOUGH

ELEMENTARY ABSTRACT ALGEBRA. By E. M. Patterson and D. E. Rutherford. (Oliver & Boyd) 13s. 6d.

This book covers most of the first year Algebra course needed for the Part IA Mathematics Tripos examination. It provides a good outline of the subject and would be a fairly good buy for the first term or even before coming to university. It is also enriched with simple examples and exercises in each chapter complete with answers at the back.

Being in the Oliver & Boyd series, it also has the advantage of being fairly cheap (13/6 in paperback) and fairly compact. However, for the algebraic enthusiast it would not be much good except for introductory reading. The subjects covered include mappings, binary operations, group theory, ring theory and vector spaces. R. J. TRIANCE

A COURSE OF HIGHER MATHEMATICS. Vol. V 'Integration and Functional Analysis'. By V. I. Smirnov (Pergamon) £6. 6s.

Imagine a hard-working Soviet technologist, (or the present editor of 'Eureka', for that matter), ploughing his way through a massive textbook of heavy analysis. Then you will be thinking of a book such as this fifth volume of Prof. Smirnov's series on Higher Mathematics. This impression is probably due to a tendency to dwell at length on relatively unimportant details and thus to obscure the basic principles of the subject. This is particularly true of the first three chapters—nearly half of the book—which are devoted to the theory of the Riemann-Stieltjes and Lebesgue integrals. There are many books on integration theory which can be recommended in preference to this.

Fortunately, the second half of the book is considerably better. After a short general chapter on metric and normed spaces, the last (fairly long) chapter gives a good description of the theory of Hilbert space, including the spectral theorem, the theory of unbounded operators and the theory of operators in l^2 and L^2. This latter half of the book is quite comparable in quality with most other books on its subject. P. G. DIXON

ITERATIVE METHODS FOR THE SOLUTION OF EQUATIONS. By J. F. Traub. (Prentice-Hall).

ANALYSIS OF VARIANCE. W. G. Guenther. (Prentice-Hall).

All books involving iterative methods suffer from a confusion of terminology and notation, and this volume by Traub is no exception. On opening the book the reader is immediately confronted by 3 pages of symbols, some of quite awesome complexity, which serve as a reference to the text. The mathematical treatment resulting is naturally rather ponderous and, furthermore, it has the unfortunate result that the work loses its appeal as a book of reference, since it must be read in full to appreciate the intricacies of any given chapter.

Despite this, the actual material is well arranged and presented with an excellent system of indexing. There is a comprehensive treatment of the four branches of iterative functions — one-point and multi-point with or without memory. Special attention is paid to the informational efficiency of the various procedures and the overall treatment is highly rigorous. In certain sections the student must however be prepared to supplement his reading from one of the recommended volumes in the bibliography.

Guenther's book on the other hand is an excellent introductory text. The approach is clear and logical, at times even over-simplified, progressing from a discussion on statistical methods involving two independent random samples to the general case, where various hypotheses are tested from n samples. Randomised blocks, latin squares, factorial experiments, and analysis of covariance are also covered, each selection containing well-chosen examples—though best tackled with the use of a desk calculator. One of the drawbacks of the book is the fact that no answers are given so that one cannot check one's procedures. Nevertheless the book is a very welcome addition to the existing literature. D. STANFORD

THE MATHEMATICAL THEORY OF OPTIMAL PROCESSES. By L. S. Pontryagin et al. Translated by D. E. Brown. (Pergamon) 80s.

The optimization problem is now recognized as the most important aspect of modern control theory. One technique for the solution of complex optimization problems is Pontryagin's Maximum principle, with which this translation deals comprehensively and in detail. This is a book for mathematicians rather than for engineers. Some knowledge of variational calculus, state-space methods and generalized co-ordinates is required.

The first chapter states the principle without proof, and discusses its use in some simple time-optimal problems; it may be read by itself as a clear and readable introductory discussion. Chapter 2 gives a detailed and very abstruse proof of the principle, and the remainder of the book is devoted to considering its application to specific cases.

It is doubtful whether this book will be of use to the engineer seeking a general introduction to the theory of optimal processes. However, the book is to be recommended to theorists and research workers as authoritive and valuable. A. BAINBRIDGE

SHORTEST PATHS. By L. A. Lyusternik. (Pergamon). 17s. 6d.

Here is a book that does little to raise the standard of elementary popular expositions; much of it is devoted to confusing the reader with fudged proofs ('We shall prove this fact, although we do not claim that our proof is by any means rigorous', p. 76) and may turn many people away from an attractive subject. It may be impossible to do otherwise using the weapons at the author's disposal, namely trigonometry without calculus, but that seems no excuse for foisting the results on the layman who needs especially the clarity and elegance that this book lacks. C. L. THOMPSON

CLASSICAL MECHANICS. By D. E. Rutherford. (Oliver and Boyd) 10s. 6d.

This is the third edition of the book in the well-known University Mathematical Text series. M. K. AYRES

AN INTRODUCTION TO MODERN MATHEMATICS. By A. Monjallon. (Oliver and Boyd). 13s. 6d.

Professor Monjallon begins with an interesting and exhaustive study of Set Theory, and on this basis he builds relations, functions, and finally Abelian groups. The book, which is a translation, is clear and readable throughout and seems ideal for post A-level sixth form reading or for perusal by freshmen on arrival at university. As it claims, the book definitely fills a gap in the existing literature and as such it is very welcome.

J. FILOCHOWSKI

ABOUT VECTORS. By Banesh Hoffmann. (Prentice-Hall). Paper 26s.

I wonder why this book was ever written. The preface tells me—'. . as much to disturb and to annoy as to instruct. . . if it should cause a re-examination of fundamentals in classroom and mathematics club it will have achieved one of its main purposes.' If it causes as much as one flickering eyelid I shall be very surprised indeed.

The first chapters are a rehash of any standard elementary text on vectors with a pedantic obsession with trivial detail. The last chapter enters the realm of tensors with a little more success, showing the relationship between vectors and tensors. Perhaps the book is suitable reading for a very bright and rather inquisitive schoolboy, but for very few others I think. J. FILOCHOWSKI

A SHORT INTRODUCTION TO NUMERICAL ANALYSIS. By Professor M. V. Wilkes. (C.U.P.). 25s. (Paper 10s. 6d.)

The book provides a more-than-adequate substitute for the lecture course. The paperback version works out at approximately one farthing per minute of lecture time. This apart, the book does provide an excellent and systematic introduction to Numerical Analysis which will be very valuable for beginners. J. FILOCHOWSKI

MATHEMATICAL INTRODUCTION TO CELESTIAL MECHANICS. By H. Pollard. (Prentice-Hall, Inc.) 42s.

The subject matter of this book belongs to the region between mathematics: 'a point of equilibrium is called L-stable if, for each positive number ϵ, there is a positive number δ such that each solution. . . which with initial position within a distance δ of the point exists for all time thereafter and never departs from this point to a distance exceeding ϵ. '; and applied mathematics: 'many years of observation indicate that the (Lagrange) points are L-stable'. Fortunately, the former quotation is more characteristic of the approach taken in this book. The proofs given are fairly rigorous and full use is made of the general methods of mechanics.

Should the reader want to find how to compute the path of Halley's comet at its next return, or to determine a satellite's orbit from a few well-timed observations, then this book is not for him. Such information, bodering on numerical analysis, is to be found elsewhere. Should the reader want to predict solar eclipses for the year 3000, then he will find that this book only scratches the surface of the complex problems of lunar theory. However, if his aim is less ambitious, but he would like to see more of this fascinating subject than is presented in the Tripos, then this book can be thoroughly recommended to him. P. G. DIXON

INTEGRAL EQUATIONS AND THEIR APPLICATIONS. By W. Pogorzelski. (Pergamon) 120s.

BOUNDARY VALUE PROBLEMS. By F. D. Gakhov. (Pergamon) 105s.

MULTIDIMENSIONAL SINGULAR INTEGRALS AND INTEGRAL EQUATIONS. By S. G. Mikhlin. (Pergamon) 80s.

A COLLECTION OF PROBLEMS IN MATHEMATICAL PHYSICS. By B. M. Budak, A. A. Samarski, and A. N. Tikhonov. (Pergamon) 80s.

One of the most important of the many improvements that have been made recently in the Cambridge applied mathematics course is a greater stress on mathematical methods, especially the more sophisticated techniques that can now be introduced earlier to students with a greater knowledge of basic pure mathematics. The publication of the above set of translations shows how far this country lags behind others in the study of powerful problem-solving techniques, many of which are, contrary to first impressions, quite essential in technological applications.

The book by Pogorzelski is just the sort of book to supplement the Part II differential equations courses. The early sections, on integral equations, are a little slow-moving for anyone who has mastered Analysis IV, but the sections on applications to partial differential equations are useful both for study and for reference. Even the Dirichlet Problem is cut down to size; the exact existence conditions are worth knowing.

Towards the end of the work boundary value problems of analytic functions—i.e. Laplace's equation in two rather than three dimensions—are considered; the study of such problems takes up most of Gakhov's book. This is for the research student specializing in applied functional analysis rather than for the applied mathematician, for though the mathematical knowledge required is no greater than for Pogorzelski, the viewpoint is that of a pure, rather than an applied, mathematician. However, it may well be of use in the solution of problems by complex variable methods, especially in theoretical plasma physics.

S. G. Mikhlin is among the most eminent Soviet applied mathematicians: this specialized work continues his earlier volumes, and contains many of his own results, but is probably of rather limited interest except to research students.

The collection of problems contains most of the Part II mathematical methods in summarized form, together with over 800 problems leading to partial differential equations. Most have their solutions outlined. It is possibly most useful as a reference work for all the standard separations of coordinates and on integral transforms, but some of the problems, for example that on the hydraulic hammer, occur less frequently in Tripos papers.

All these books are rather expensive for the undergraduate; the first and last, however, should be in every college library.

C. J. MYERSCOUGH

VECTOR SPACES OF FINITE DIMENSION. By G. C. Shephard. (Oliver & Boyd) 13s. 6d.

Now that a more abstract approach to vector space theory has been adopted in Part IA Algebra, there is a need for a textbook which covers the material of the course from this angle. Shephard's book fulfils this need and at 13s. 6d. (in paperback) is certainly a good buy.

Discussion of determinants and matrices is relegated to the end of the book, so some previous familiarity with these topics is essential to appreciate the motivation behind the abstract theory. However this treatment gives a greater insight into such properties as rank and equivalence.

In general the book is clear and thorough, although there is one bad logical error—in extending the definition of direct sum to a family of subspaces U, the condition: -

$$U_i \cap U_j = 0 \text{ for all } i, j; i \neq j$$

is not sufficient for independence. D. H. D. WARREN

FUNCTIONS OF REAL VARIABLES. By R. Cooper. (Van Nostrand: The New University Mathematics Series) 21s. (paperback)

This book covers a variety of topics in analysis, with an emphasis on real-valued functions of several real variables.

The first three chapters give a concise treatment of numbers, point sets, sequences, series, continuity and derivatives. Functions of several variables are then introduced and integration is considered through the Riemann Integral over a general interval. After some further results on series and definitions of the elementary transcendental functions, the remainder of the book is devoted to some special topics: the Gamma function; Stokes' Theorem and the Divergence Theorem; differential equations; Legendre polynomials; Fourier series.

The book is well-written and, while not suitable as a sole textbook for Part IA Analysis, would be a worthwhile purchase for the theory behind much of the Part IA Mathematical Methods course. Good value at 21s. D. H. D. WARREN

ELEMENTS OF THE THEORY OF PROBABILITY. By Emile Borel (translated by J. E. Freund)

This very interesting book presents probability theory in the light of many fascinating problems. Anyone suffering from too much pure mathematical probability would be healed by looking through some of the ways in which the theory is used in this book. The presentation of the book is good and no signs of its translation are evident. Among his problems, Borel discusses such matters as large numbers of trials in 'heads and tails', roulette, 'the law of chance', geometrical probabilities, the needle problem, arbitrary functions, errors, causes, twins, absolutely normal numbers and psychological games. P. D. COOPER

GENERALISED FUNCTIONS AND DIRECT OPERATIONAL METHODS. By T. P. G. Liverman.

This book builds up the theory and practices of generalised functions on three distinct levels. Any person with an idea of undergraduate mathematics could grasp concepts presented here, but he must beware all the starred sections for they are heavy going.

I cannot comment further on the higher level approaches but I found the unstarred sections interesting, even though it took time for basic ideas to sink in. Do not be deterred by the formidable appearance. P. D. COOPER

SPECIAL RELATIVITY. By W. Rindler. (2nd edition; Oliver & Boyd) 13s. 6d. (paperback: 10s. 6d.)

This small book has been considerably revised. The subject is now approached initially from a much more physical viewpoint; the 'unity of physics' is stressed. This also happens, in the later chapters, (but to a far lesser extent). Chapters I-IV are now an excellent and readable introduction to the subject while the rest of the book, in particular the appendix on general tensors, is still over-compact and difficult to follow. In the preface, the author makes (implicitly) the assumption that the most elegant presentation is the clearest.

Generally, where the book has been reviewed it has greatly improved, but this has happened in far too few places. A. KALETZKY

SUCCESSIVE APPROXIMATIONS. By N. Ya. Vilenkin. (Popular Lectures in Mathematics, Vol. 15; Pergamon) 15s.

The pure mathematician who seeks a rigorous work on numerical problem-solving will be disappointed here. So too, unfortunately, will anyone who wants anything more than an elementary introduction to the subject; the book does not get as far as dealing with Horner's method, for example. And fifteen shillings is dear for a 'popular' book of only 70 pages.

On the credit side, the style of writing is very readable, and the numerical methods that are included are described perfectly clearly. The book contains a list of problems, with answers provided. Nevertheless, this is a book for the sixth form rather than university, and for engineers rather than mathematicians. P. J. BUSSEY

MAGNETOHYDRODYNAMICS. By A. Jeffrey. (Oliver & Boyd) 13s. 6d.

MHD is such a diverse and rapidly expanding subject that every author inevitably has his own pet topic—in this case, about half the book is devoted to the study of MHD waves. Considering that the subject is developed assuming no previous knowledge of fluid mechanics, and the whole development is rather slow-moving, it is not surprising that few other topics are described in detail in so short a book. Fortunately there are a number of other inexpensive books on the subject available (e.g. Shercliff, Ferraro & Plumpton); together they give a fairly comprehensive view. C. J. MYERSCOUGH

INTRODUCTION TO MEASURE AND PROBABILITY. By J. F. C. Kingman and S. J. Taylor (Cambridge University Press) 70s.

This good book has two particular qualities. It is very readable and it is quite self-contained. The first two chapters contain a useful amount of set theory and point set topology. Next the theory of measure and Lebesgue integrations are developed in a very general fashion. The material is presented with many explanatory asides so one does not lose sight of the wood for the details of the trees. The middle four chapters give an introduction to functional analysis and the last part of the book is devoted to probability theory.

It is not however a book to dip into; the notation, though very good, might be annoying if one had not read a largish part of the book. This is no hardship for, as I have already said, it reads very easily. It is rigorous throughout but marred by a fair number of misprints which will no doubt disappear from the later editions. G. D. PATERSON

(Continued on p. 26)

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