Eureka Editor
archim-eureka@srcf.net
The Archimedeans
Centre for Mathematical Sciences
Wilberforce Road
Cambridge CB3 0WA
United Kingdom

Published by The Archimedeans, the mathematics student society of the University of Cambridge

Thanks to the Betty & Gordon Moore Library, Cambridge

EUREKA

No. 21

OCTOBER, 1958

THE JOURNAL OF THE ARCHIMEDEANS

The Cambridge University Mathematical Society; Junior Branch of the Mathematical Association

Contents

Contributions and other communications should be addressed to:

The Editor, "Eureka,"
The Arts School,
Bene't Street,
Cambridge,
England.

Editorial

It is only natural for a university student to wonder what effect—if any—his studies have had on the working of his mind; whether, as one hopes, they train him to think clearly and without prejudice or whether, as cynics aver, after the hectic and unedifying scramble for good examination results his mind slips back to its pristine state. I am optimistic, and here indicate some of the results that I believe this training can produce.

Mathematics is *par excellence* the study where pure logic is applied, and it is not possible to obtain a good class in any examination above Part I unless one not merely realises this but is competent at it. Now logic, like science, like art, like philosophy tends to delude its adherents into the belief that it is the ideal method of obtaining truth, to which others ought to be subordinated. Thus it is not uncommon to find mathematical students discussing realms of thought far from the cold and glittering peaks of pure mathematics, yet still applying pure logic—the one way they have really learnt of proceeding from one truth to another. Assent would not be given by all to this observation, even by all mathematicians, but it seems to be true in my experience.

If—as I have hinted—logical methods are not suitable for all problems, do we conclude that this state of affairs is undesirable? I do not think so. Firstly (and I suppose this is due to my own mathematical bias) I feel that pure logic is a technique that educated people should be able to understand and to use, though this is far from sufficient; logic *unaided* proving remarkably barren in most fields of enquiry. But secondly I consider that this emphasis gives us a distinctive contribution to make to discussions in which we are engaged. For if a logical tangle appears to arrive at a contradiction, and the general opinion of assembled company is that an impasse has been reached, the mathematician should be the first to analyse it, to show that the fault lies not in the logic but in its application, and to bring to light assumptions subconsciously made in the course of the argument, which need by no means be made. You may feel that in undergraduate discussions this is too much to ask, but I imagine that in a more everyday atmosphere at home you will be much better equipped for this sort of thinking than others. Following on from this, the mathematician should be expert at writing and thinking concisely, and at reducing problems to their essential elements—after all, one has practice abbreviating lecture notes to the point of unintelligibility! There is, to my mind, positive evidence in minutes of society meetings, correspondence, and in articles in this magazine that we do this.

Finally, as well as becoming good at solving abstract problems, we should be gaining confidence in inventing them. I do not refer here to problems within mathematics, but well outside in ordinary walks of life. We should feel an urge to examine the assumptions on which policies are based with whose working out we are charged, and communicate this questioning spirit to others. I have from time to time wondered whether it is our success at asking ourselves abstract questions about the purpose of life in general that is the cause of the fact (which I certainly observe) that a very high proportion of mathematicians seem to be keenly Christian and concerned about the life they lead, while most of the rest have come to a different answer to the same problem and thrown their energies into the Socialist movement.

The Archimedeans

OUR programme card for this year is the fullest we have ever had. There are more social events and other activities not directly connected with mathematics, and we also have a particularly distinguished list of speakers for a larger number of evening meetings.

An important meeting will be that which we are holding in November together with the Cambridge Philosophical Society, when Professor Synge is coming over from Dublin to deliver the Larmor Memorial Lecture. He will be talking about the principle of relativity, and as with most of our speakers his aim will be to try and make his lecture intelligible to freshmen while at the same time of interest to those of us who think we know a bit more about maths.

Because of their popularity we are also holding more tea meetings this year than last. Research students will be talking about their work in widely different fields, from electronics to genetics, while to start the year Thurston Dart is giving a talk on mathematics and music.

Towards the end of this term we are, for the first time in some years, to have a Society Dinner. Our principal guests will be professors from Oxford and London as well as from Cambridge. We particularly hope that as many as possible of our members, both senior and undergraduate, will come and help to make the occasion a success. Other social activities include a Coming-up Dance, and Archimedes' Bathday Party. In addition regular meetings will be held of the Bridge Group, which will concentrate on teaching novices as well as providing games for more experienced players, and of the Music Group, which intends to play a few rather more unusual records this year.

The Committee has tried to arrange as interesting and varied a programme as possible. It is now up to you to give support to your Society! P. V. LANDSHOFF.

The Explorer's Problem

by

I. C. PYLE

A CERTAIN lorry can carry enough petrol to go 500 miles. Is it possible for this lorry to cross a desert (*a*) 1,000 miles, (*b*) 1,500 miles, (*c*) 2,000 miles wide, and if so, what is the minimum amount of petrol that it needs? It can be assumed that there are infinite supplies of petrol at the edges of the desert, but no others: the lorry must be used to build up its own refuelling stations en route. Losses by evaporation, etc. may be neglected.

For simplicity, we will call the maximum amount of petrol the lorry can carry one load, and the distance it can go on this unit distance. The refuelling stations will be called caches.

At each stage, the lorry must make a number of journeys from a given cache, building up the next one. This number of journeys must clearly be odd, or the lorry would not be able to progress to the next stage.

For the transfer of petrol between caches a given distance apart, the most efficient method is that in which the lorry travels the shortest distance. Therefore, if the transfer can be carried out in three journeys it would be inefficient to make five journeys, or more.

However, there is a maximum amount of petrol which can be handled in a given number of journeys, for each time the lorry departs from the initial cache, it can only carry one load. Therefore $n + 1$ loads can be taken if there are $2n + 1$ journeys. Of this, some is used by the lorry in travelling, and the remainder is delivered to the next cache.

Since the transfer is more efficient for a smaller number of journeys, the greatest possible use must be made of such transfers.

We start by considering the simplest, and most efficient stage: one journey. This can take the lorry distance 1, and requires 1 load.

Therefore it must be the last stage of the crossing of the desert, and requires a cache containing one load at a distance 1 from the far side of the desert.

Since no further distance can be covered with one journey, we next consider three journeys. These can handle up to 2 loads. One load must be delivered at the next cache, therefore the distance between the caches must be $\frac{1}{3}$ or less. As explained above, for greatest efficiency, we choose the maximum, and require a cache containing 2 loads, at a distance $1 + \frac{1}{3}$ from the far side of the desert.

Similarly with five journeys, up to 3 loads can be handled, and 2 loads must be delivered at the next cache. Thus the maximum distance which can be covered is $\frac{1}{5}$. Therefore we require a cache containing 3 loads at a distance $1 + \frac{1}{3} + \frac{1}{5}$ from the far side of the desert.

It is easy now to see that in general, if the lorry makes $2n + 1$ journeys, it can handle $n + 1$ loads, and must deliver n, at a distance $1/(2n + 1)$. Hence a cache containing $n + 1$ loads is required at

a distance $s_n = \sum_{r=0}^{n} \frac{1}{2r+1}$ from the far side of the desert.

This sum diverges as $n \to \infty$, therefore *any* desert can be crossed, no matter how wide.

To discover the amount of petrol needed, we must take partial sums of the series, and determine when the actual width of the desert is reached.

(*a*) For width 2 units (1,000 miles), we find

$s_6 = 1{\cdot}955133,$

$s_7 = 2{\cdot}021800.$

Therefore the first actual cache must contain 7 loads, and be situated 1·955133 units from the far side, i.e. 0·044867 from the near side. The lorry must make fifteen journeys to this cache, for which it uses 0·6730 loads of petrol.

Therefore the total amount of petrol needed is 7·6730 loads.

(*b*) For width 3 units (1,500 miles),

$s_{55} = 2{\cdot}994438,$

$s_{56} = 3{\cdot}003288.$

The first actual cache must contain 56 loads, and be 2·994438 from the far side, i.e. 0·005562 units from the near side. The lorry must make 113 journeys to it, using 0·6285 loads of petrol.

The total needed is therefore 56·6285 loads.

(*c*) For width 4, an asymptotic approximation is obviously called for:

$$s_n = 0{\cdot}63518 + \tfrac{1}{2}\log_e(2n + 1),$$

and $$s_n = 4 \text{ for } n = 417{\cdot}92.$$

Thus well over 400 loads will be required.

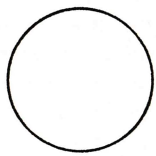
Why not buy an aeroplane?

[We are not convinced that the above method is the most efficient possible, but have yet to see a better one—*Ed.*]

An Elevation Puzzle

BELOW are shown the front elevation and plan of a mathematical figure. What is the side elevation?

Front Elevation



Plan

(For solution see page 29)

Our Founder

by

H. T. CROFT

Archimedes of Syracuse was the son of Pheidias the astronomer, and on intimate terms with, if not related to, King Hieron and his son Gelon. He spent some of his life in Alexandria, and was friendly with Conon of Samos and Eratosthenes; then returned to Syracuse for a life devoted to mathematical research. He perished in 212 B.C. (at age 75, according to Tzetzes) in the sack of Syracuse.

Stories of other details of his life, culled from many sources, are somewhat dubious. No authenticated picture remains, in spite of three (totally different) purported portraitures in classical works of the last century. The only contemporary biography is not extant.

Tales of his preoccupied abstraction—drawing diagrams in ashes, or in oil when anointing himself, and forgetfulness of food—remind us irresistibly of Newton's going out in a fit of absentmindedness without his trousers. He died as he had lived, deep in mathematical contemplation. Several authors give variously garbled accounts, the most picturesque being that, though Marcellus the Roman commander wished him to be spared, a common soldier, enraged by the great man's request to "Stand away, fellow, from my diagram," dispatched him. As he had asked, his discovery of the surfaces of the sphere and cylinder was depicted on his tombstone, which was later found in a dilapidated state and restored by Cicero when quaestor in Sicily.

His mechanical achievements include the water-screw, invented in Egypt for irrigational purposes and used for pumping from mines or ship-holds, and a very accurate model of the planetary system demonstrating eclipses. Some of his inventions were very effective during the siege of Syracuse—catapults of variable range and other machines discharging showers of missiles, and crane-like grappling contrivances which seized the prows of ships and thus played "pitch-and-toss" with them. The Romans were in such abject terror that "if they did but see a piece of rope or wood projecting above the wall, they would cry 'there it is again,' declaring that Archimedes was setting some engine in motion against them, and would turn their backs and run away, insomuch that Marcellus desisted from all conflicts and assaults, putting all his hope in a long siege" (Plutarch). The story that he fired the Roman ships by use of concave burning-glasses and mirrors is very doubtful, being first recorded in Lucian, 300 years later.

When Hieron asked for a practical demonstration of a great weight moved by a small force, in connection with his famous utterance "δός μοι ποῦ στῶ και κινῶ τὴν γῆν" (Give me a place to stand on, and I can move the Earth), he drew a loaded ship safely and smoothly along with a compound pulley or, according to another account, a helix, a machine with a cogwheel with oblique teeth. Hieron thereupon declared that "from that day forth Archimedes was to be believed in everything that he might say."

Born just before the death of Euclid and 30 years senior to Apollonius of Perga, the "Great Geometer" and last of the three great mathematicians of antiquity, he wrote works with a larger proportion of originality. "It is not possible to find in geometry more difficult and troublesome questions or more simple and lucid explanations." Like most of the ancients, he left little clue as to his method of discovery. He seems "as it were of set purpose to have covered up the traces of his investigation as if he had grudged posterity the secret of his method of inquiry while he wished to extort from them assent to his results" (Wallis). But a manuscript of the "Methods of mechanical theorems," discovered in 1906 in Constantinople and addressed to Eratosthenes, lifts the veil a little.

Other works entitled "On the equilibrium of planes," "On the quadrature of the parabola," "On conoids and spheroids," "On floating bodies," "On the measurement of a circle," "The Sand-reckoner" and a collection of lemmas indirectly due to him are still extant. Lost works are thought to refer to polyhedra, balances and levers, centres of gravity, the calendar, optics, water-clocks and a work entitled "On sphere-making." Arabian writers attribute other works to him.

His main achievements were: quadrature of the parabola, finding surfaces and volumes of spheres, segments of spheres and segments of quadrics of revolution, the approximation $3\frac{1}{7} > \pi > 3\frac{10}{71}$, the invention of a number-scale up to 10 to the power 8.10^{10} (in the Sand-reckoner), geometrical solution of some cubic equations, a method of finding square roots of non-squares, and the whole science of hydrostatics even up to determining the positions of equilibrium and stability of floating segments of a paraboloid. He was also much occupied by astronomy—Livy calls him *"unicus spectator caeli siderumque."* He is further credited with authorship of the "cattle-problem," which involves eight unknowns and the solution of which has 12 or 206545 digits according to how an ambiguous statement is interpreted. The *"loculus Archimedius,"* a puzzle of 14 shapes fitting together to form a square, is now thought due to him, although the phrase *"πρόβλημα 'Αρχιμήδειον"* was simply a proverbial expression for something very difficult.

He regarded his ingenious mechanical inventions simply as "diversions of geometry at play" and "he possessed so high a spirit, so profound a soul, and such treasures of scientific knowledge that, though these inventions had obtained for him the renown of more than human sagacity, he yet would not deign to leave behind him any written work on such subjects, but, regarding as ignoble and sordid the business of mechanics and every sort of art which is directed to use and profit, he placed his whole ambition in those speculations in whose beauty and subtlety there is no admixture of the common needs of life" (Plutarch).

Contributions

MANY thanks to our numerous contributors, who make the magazine what it is. They will be paid, as announced, at 10s. per printed page. I cannot guarantee the next editor's policy, but will be very surprised if some such system is not maintained. It is of great assistance to us if contributions are sent early, typed or neatly written, on one side of the paper only, please.

A Function with an Infinity of Saddle Points

by

MARTIN FIELDHOUSE

THE hypergeometric function, $M(a, b; x)$, is defined by the infinite series

$$1 + \frac{a.x}{b.1!} + \frac{a(a+1)x^2}{b(b+1)2!} + \frac{a(a+1)(a+2)x^3}{b(b+1)(b+2)3!} + \ldots$$

This series is convergent for all a, b and x except $b = -n$ ($n = 0, 1, 2 \ldots$) when it is not defined. $M(a, b; x)$ is an analytic function of the three variables a, b and x: that is, it is both continuous and differentiable everywhere.

This function may be thought of as being more basic than either the exponential or Bessel functions for these are special and limiting cases of it. Hypergeometric functions are important in many branches of applied mathematics from aircraft design to nuclear physics.

Of special interest in the theory of the hypergeometric function is the zero-surface defined by

$$M(a, b; x) = 0 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (1)$$

We need only consider the surface for x positive, because the relation

$$M(a, b; x) = e^x M(b - a, b; -x) \quad \ldots \quad \ldots \qquad (2)$$

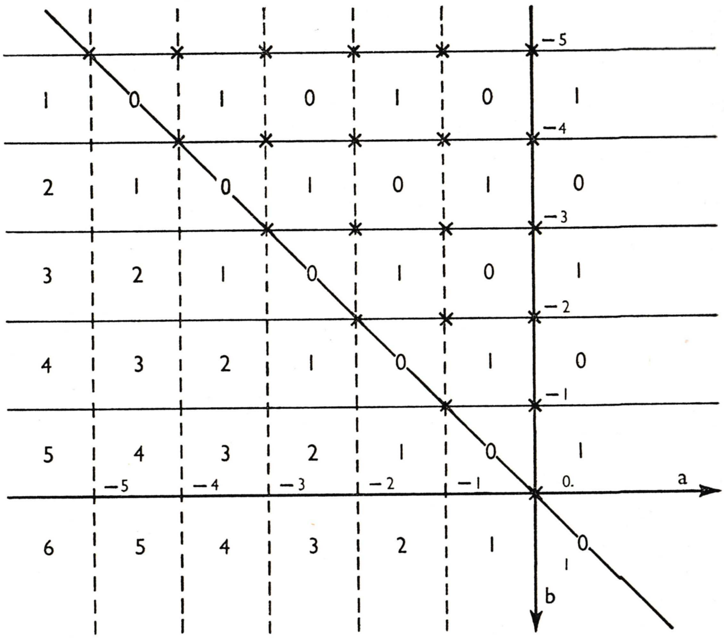
shows that the surface for x negative is merely a distorted reflection of the surface for x positive.

It can be shown that, given a and b, the function has only a finite number, N, of positive values of x satisfying (1). The distribution of N is shown in the figure.

Note (i) On the plane $a = b$, $M = e^x$,
(ii) On the plane $a = 0$, $M = 1$,
(iii) On the planes $b = -n$, M is not defined,
(iv) Near the points marked with crosses, M can take any value whatsoever.

A qualitative impression of the zero-surface may be obtained as follows. When $x = 0$, $M = 1$ and the surface is asymptotically the lines $b = -n$. When x is very large positive, the function is positive in regions where N is even and negative where N is odd.

As x increases and tends to $+\infty$, the parts of the lines $b = -n$, to the left of $a = b$ in the figure sweep round anticlockwise until they become very nearly the lines, $a = -n$. Thus the zero-surface in this region is a set of overlapping surfaces which become approximately the planes $a = -n$ when x is large.



To the right of $a = b$ in the figure, the behaviour is quite different. The zero-surface exists only in alternate squares and in the alternate strips to the right of $a = 0$. Yet in this region too, as $x \to +\infty$, the zero-surface tends to the planes $a = -n$, always being just on the sides of these planes where it exists. Any cross-section, $x = k$, of the zero-surface contains the points marked with crosses asymptotically. Because of the continuity of the function, this state of affairs can only be realised if we assume the existence of a saddle-point of the zero-surface in each of the alternate squares in the sector bounded by $a = 0$, $a = b$. An infinity of saddle-points!

To check this assumption, a program was written recently for the computer, EDSAC II, to search for these saddle-points and to tabulate them. EDSAC succeeded in finding the first hundred or so. As a point of interest, they all lie fairly close to the plane

$$3x + 2a + b = 2.$$

Mathematical Association

President: Professor M. H. A. Newman, F.R.S.

The Mathematical Association, which was founded in 1871 as the *Association for the Improvement of Geometrical Teaching*, aims not only at the promotion of its original object, but at bringing within its purview all branches of elementary mathematics.

The subscription to the Association is 21s. per annum: to encourage students, and those who have recently completed their training, the rules of the Association provide for junior membership for two years at an annual subscription of 10s. 6d. Full particulars can be had from The Mathematical Association, Gordon House, Gordon Square, London, W.C.1.

The *Mathematical Gazette* is the journal of the Association. It is published four times a year and deals with mathematical topics of general interest.

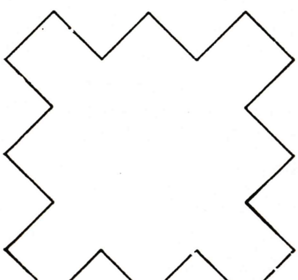
The Cross-Stitch Curve

by

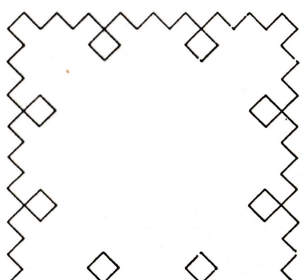
M. W. Birch

MANY readers will be familiar with the "snowflake" curve made up by building up small triangles onto the sides of a large triangle and smaller triangles onto the resulting figure and so on. I wonder how many have tried the same thing with squares. The resulting figure, though degenerate, is no less interesting.

We start with a square T_1. We trisect each of its edges, and on the middle part of each edge erect another square, thus arriving at T_2. We now trisect each of the edges of T_2 (both those originally belonging to T_1 and the new edges) and erect another set of squares as before, so obtaining T_3.



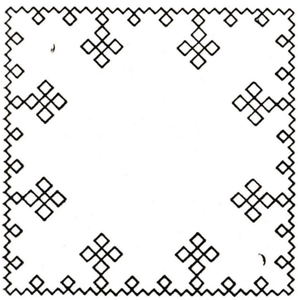
T_2



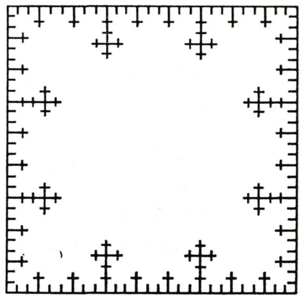
T_3

The process may be continued indefinitely as none of the small squares overlap, though they may touch corner to corner.

It may easily be proved by induction that the area of T_n is $2 - (\frac{5}{9})^{n-1}$, and the perimeter $(\frac{5}{3})^{n-1}$. It follows that as n increases T_n fills up the large square double the original square, while the perimeter increases without limit. The curve eventually appears at first glance like the figure T (it is of course more detailed), with its "cross-stitches" along the diagonals of the isolated squares. In fact each straight part of the curve is at 45° to these directions.



T_4



T

Lebesgue's Minimal Problem

by

H. S. M. Coxeter

THIS will be the new name for "Besicovitch's Minimal Problem" in the new reprinting of Rouse Ball's *Mathematical Recreations and Essays* (London, 1939, p. 99). In fact, Professor Besicovitch disclaimed his alleged authorship soon after the appearance of the eleventh edition. Lebesgue's statement of the problem is quoted by Julius Pál, *Ueber ein elementares Variationsproblem,* K. Danske Videnskabernes Selskab., Math.-fys. Medd. 3 · 2 (1920), p. 4. Anthony Edwards's challenge (EUREKA, October 1957, p. 26) is answered by observing that any given plane figure of unit diameter can be covered by a regular hexagon of side $1/\sqrt{3}$, whose area is $\frac{1}{2}\sqrt{3} = 0{\cdot}8660$. Pál (p. 18) improved this by truncating two alternate corners of the hexagon so as to reduce the area to $2 - 2/\sqrt{3} = 0{\cdot}8454$. A further slight improvement was made in 1954 by R. P. C. Caldwell of the University of Illinois; he cut off another piece to leave the area 0·8444. The least upper bound is still unknown.

Problems Drive, 1958

1. Give the next term of the following four sequences:—

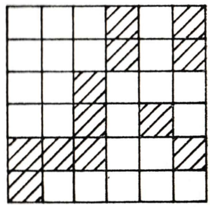
   (i) 1, -1, 1, 13, 41, 91, 169, . . .

   (ii) 3, 5, 11, 17, 31, 41, 59, . . .

   (iii) (2, 3), (1, 7), (1, 4), (1, 8), (2, 5), . . .

   (iv) 1, 2, 2, 4, 2, 4, 2, 4, 6, 2, 6, 4, 2, 4, 6, 6, 2, . . .

2. Show how to cut the figure given into just four pieces of the same size and shape, cutting only along the lines, so that each piece contains the same number of shaded squares.



3. A man stays at a desolate hotel for 26 weeks. He has no money but has an endless gold chain of 182 links. How many links of the chain must be broken so that the client may pay the landlord one link (possibly broken) each day? (On any one day it is quite permissible for, say, the man to give the hotelier 13 links in exchange for a string of 12 links.)

4. The four boys discovered a dish full of peanuts. Peter at once ate 3 and then Quentin and Ralph each consumed $\frac{1}{9}$, and Stephen $\frac{3}{11}$, of the remainder. Quentin now disposed of 2 more while Peter, Ralph and Stephen divided $\frac{1}{2}$ of what were then left so that Peter got one more than each of Ralph and Stephen. Next Peter and Ralph took 5 each, following which Quentin, Ralph and Stephen shared out $\frac{5}{6}$ of the rest in the proportion 3 : 6 : 10. Ralph greedily ate a further 1, and then Peter, Quentin and Ralph respectively took $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{9}$ of the remainder. They now discovered to their horror that there were only 4 peanuts left, which they shared out, receiving 1 each.

   How many nuts were there at first, and how many did each boy have?

5. When Michael came home on leave from Germany to see his girl friend Mary he found her playing poker in the drawing room with her nine cousins: Angela, Joyce, Esther, Mark, John, Phillip, James, Kate and William.

   When the poker was over the cousins discreetly left the room.

Joyce did not leave until Esther had gone. Two brothers, one of whom was Mark, left together leaving only two men behind. Phillip left before Joyce did. When Angela went she left an odd number of people behind her. No two ladies left consecutively. The last person to leave was a man and Esther was not the first lady to leave. William left before John, who was not Mark's brother. Phillip, who was not the first man to leave, left between two ladies. James did not leave before William.

Who were the last two people left in the room?

6. For all x, y, z, $(xy)z = x(yz)$ is written xyz

xx is written x^2, xxx x^3, etc.

$x1 = 1x = x$

but $xy \neq yx$ in general.

(*a*) If $f^3 = g^4 = 1$ and $fg = g^3 f$ prove that $f^2 g^2 = g^2 f^2$

(*b*) If $f^3 = g^4 = h^2 = 1$ and $fg = hf^2$, $g^3 h = f^2 g$ prove that $f^2 g^2 h^2 = hg^3 f$.

7. What is the significance of the following numbers?

(i) 31,536,000.

(ii) 98696044011.

(iii) 1,125,899,906,842,624.

(iv) 0588235294117647.

8. Show how to obtain at the same time the given quantities of beer, one in each of the given measures (but in any order), using only the measures stated, with an unlimited amount of beer available in large casks.

(i) Obtain 1, 6, 7 pints, using 3 measures, of 6, 10, 15 pints respectively.

(ii) Obtain 1, 2, 3, 4 pints using 4 measures, of 4, 6, 9, 12 pints respectively.

(iii) Obtain 1, 3, 6, 8, 9 pints using 5 measures, of 6, 9, 12, 15, 21 pints respectively.

9. Find integral solutions of

(i) $41x - 17y = 5$.

(ii) $31x - 19y = 7$.

(iii) $131x - 219y = 3$.

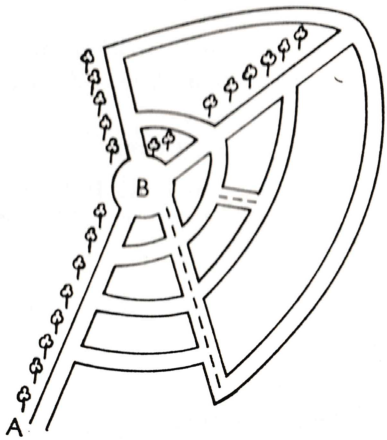
(iv) $36458x - 14667y = 13$.

10. Using the digits 1, 2, 3, 4, 5 in that order, exactly once each, and any of the usual arithmetical signs, form the numbers:

(i) 100, (ii) $3\frac{1}{7}$, (iii) 32769.

Example:—$0 = (1 \times 2) - 3 - 4 + 5$. Or $1^{23} + 4 - 5$.

11. In how many ways may a man walk from A to B? He may not retrace his step. The three tree-lined roads may only be traversed towards B. The ring roads may only be walked along in an anticlockwise direction. The dotted roads may be traversed either way.



12. Prove that there is no solution to the problem:—

"Find three primes such that the second is 16 + the square of the first, and the third is 24 + the square of the first."

Postal Subscriptions and Back Numbers

For the benefit of persons not resident in Cambridge we have a postal subscription service. You can enrol as a permanent subscriber and either pay for each issue on receipt or, by advancing 10s. or more, receive future issues as published at 25 per cent. discount, with notification when credit has expired.

Copies of Eureka Nos. 11, 13 to 17 (6d. each), 18 (1s.), 19 (1s. 6d.) and 20 (2s.) are still available (postage 2d. extra on each copy). We should also like to buy back any old copies of Nos. 1 to 10 which are no longer required.

Cheques, postal orders, etc., should be made payable to "The Business Manager, Eureka" and addressed to The Arts School, Bene't Street, Cambridge.

Brains Trust

A Brains Trust on the teaching of mathematics in Cambridge was held during the Lent term. The panel consisted of Dr. D. R. Taunt and Dr. I. Proudman representing the lecturers of the faculty, Mr. A. L. Brown representing the research students and Mr. F. P. Bretherton representing the undergraduates.

The first question asked for comments on the setting of essays for supervisions. The whole panel was agreed that no substantial improvements could be expected by such an innovation which had in the past been tried.

In answer to a searching question dealing with the aims of the lecture system the general feeling was that lectures provided the informality necessary for a full understanding of the subject. Lecturers could be less precise in the interests of clarity than textbook writers, although all too often lectures failed to come up to these standards. It was suggested that printed notes should take the place of the traditional note-taking technique, and that a detailed syllabus containing precise references should be circulated before the course began. The two lecturers both thought that the first proposal would take any life out of the lectures, but Dr. Proudman was prepared to experiment with the second suggestion. Dr. Smithies, from the floor, regretted the passing of the habit of undergraduates handing in solutions of problems set by the lecturer, as this afforded a means of the lecturer keeping in touch with his audience.

The next question dealt with the construction of the Tripos and suggested a redistribution of the syllabus over the three years available. The opinions of the panel made it clear that the Faculty Board was by no means complacent, but the audience were reminded that for many people Part I served as a springboard to another Tripos. Dr. Taunt felt that the Preliminary examination was too often taken after only one year, but Dr. Proudman modified this criticism and recommended that some students should take three years over the Preliminary examination and Part II of the Tripos. The panel welcomed the new Part II with its tendency towards specialisation and suggested that this fostered the spirit of intelligent inquiry. We were reminded, however, that new proposals for the organisation of the Tripos had recently been published.

A question on the supervision system stimulated strong comment. It was firmly maintained that supervisors should correct work before the supervision, and that they should not be called upon to supervise more than two students at a time. There was considerable diversity of opinion on the necessity for and regularity of changing supervisors. On the question of organising supervisions on a university basis the panel was divided. Dr. Taunt and Mr. Bretherton pointed out the importance of contact between the senior and junior members of a college, and felt that much would be lost if supervisions were deprived of their individual collegiate flavour. Dr. Proudman and Mr. Brown objectively considered the plight of the members of the smaller colleges with few senior members of the faculty on the foundation. Dr. Proudman's preference was to supervise students who attended his lectures rather than members of his college who might do so.

Whilst the panel were all conscious of the existence of "lecturing technique" they were unanimously against any formal course of instruction for would-be lecturers—Dr. Taunt making the point that seminars and other informal gatherings served to give them much valuable experience. Mr. Bretherton tactfully suggested that in this respect Cambridge had fallen rather behind other universities, thus emphasising the importance of this aspect of university teaching from the point of view of the undergraduate. The suggestion that lecturers should hold supplementary discussion classes was viewed sceptically by the panel—at any rate as far as Part II of the Tripos was concerned, although Dr. Batchelor, from the floor, declared himself quite willing to experiment with the idea.

The spirit of reform prompted a question on the possibility of the introduction of a course on the History of Mathematics, but all the panel, while in sympathy with the suggestion, found difficulty in seeing how such a course could in practice be embodied in the Tripos.

The meeting was closed after just over two hours of stimulating discussion which all present felt might well result in a certain amount of inspired reform.

D. M. B.

NOTE:—Duplicated copies of a full report on the Brains Trust, and of a short list of suggestions arising out of it are available on application at the Faculty Office (Room 2, The Arts School).

Definitions

One-to-one Correspondence: Personal letters.
Bar Magnet: Director of a Brewery.
Complex Transformation: A busy night at the Pentacle Club.
Conservative Fields: The grounds of Chartwell.
Lamb's Dynamics: Springtime.
Projective Space: A pause by a ventriloquist.
Principle of Least Action: Laziness.
Stream Function: A party on the Cam.

Trio

by

S. Simons

In this article I am going to put before you three problems. Certainly none of the three is difficult. On the other hand one might well search for a considerable time before finding the solutions and, if so, might justifiably be annoyed at not having discovered them rather more quickly. The second problem was published in the "American Mathematical Monthly."

1st Movement.

The only real interest in this problem is that Lewis Carroll was unable to solve it:—

A number p is the sum of two (positive integral) squares. Does $2p$ necessarily have this property?

2nd Movement.

f and g are two positive continuous functions of x in $[1, \infty)$. Their orders of magnitude are such that $\int_1^\infty f\,dx$ diverges and $\int_1^\infty fg\,dx$ converges. Does it follow from this that $\int_1^\infty \frac{f}{g}\,dx$ diverges?

h is continuous in $[1, \infty)$ and $\int_1^\infty h\,dx$ converges. Does it follow from this that $\int_1^\infty \frac{1}{x^2 h}\,dx$ diverges?

3rd Movement.

What is the least possible total length of a curve C on a unit sphere which intersects (i.e. has a point in common with) every great circle on the sphere? We allow C to have any number of disjoint rectifiable sections. What is really at stake is whether we can get the total length of C as small as we please by making each "bit" very small and the number of "bits" very large.

The reader should now industriously go and solve these problems. However, for those who are unable, through circumstances either within or beyond their control, to do this I shall outline the solutions.

1st Movement (Scherzo).

$2(a^2 + b^2) = (a + b)^2 + (a - b)^2$. Hence, if p can be expressed as the sum of *different* squares, then so can $2p$. If, on the other hand, the only expression of p as the sum of squares is $a^2 + a^2$, then it is easy to show that $2p$ is the sum of squares if, and only if a^2 itself is the sum of squares.

2nd Movement (Minuet).

If $\int_1^\infty \frac{f}{g}\,dx$ converged then so also would $\int_1^\infty \left(fg + \frac{f}{g}\right)dx$ or $\int_1^\infty f\left(g + \frac{1}{g}\right)dx$. This is impossible, for $\int_1^\infty f\,dx$ diverges and $g + 1/g \geqslant 2$ $(g > 0)$. Consequently $\int_1^\infty \frac{f}{g}\,dx$ must diverge.

(*Trio.*) If we restrict h to be positive, then the answer is "yes" as in the minuet, but here the theme is not just a variation on that of the minuet. The theme of the "second subject" is completely different. Suppose we let h take both positive and negative values. However small we make the order of h at ∞, it is still possible that $\int_1^\infty \frac{1}{x^2h}\,dx$ converges. (As the integrand is not always finite, we must interpret the integral in the improper sense.)

Suppose k is positive and $\to 0$ monotonically as $x \to \infty$. We require as a convergence condition that $h = O\,(k)$ at ∞. Then we can find an h satisfying all these conditions whatever the function k. Take h to be $k \sin^4 y$, where y is a continuous increasing function of x. By making the order of y at ∞ sufficiently large we can ensure the convergence of $\int_1^\infty \frac{1}{x^2h}\,dx$.

3rd Movement (Finale).

The solution to this is far easier than most people expect:—

To any great circle G on the sphere there corresponds a diameter of the sphere Gα in the obvious way (Fig. 1).

Consider the set S of great circles which pass through a point P of the sphere. Sα, the corresponding set of diameters, comprises the equatorial plane of P (Fig 2).

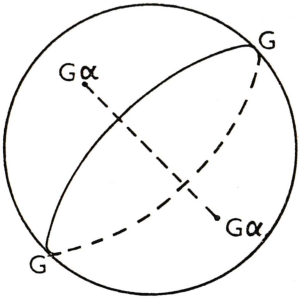


FIG. 1.

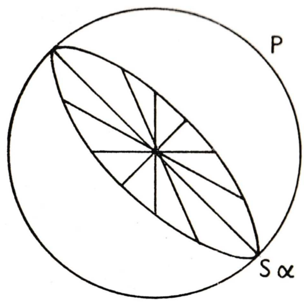


FIG. 2.

Now we can imagine an "element of length" dl on the sphere as made up of an aggregate of points P. The set of great circles intersecting dl corresponds to a "double melon slice" of diameters, subtending a total solid angle $4dl$ at the centre (Fig. 3). If, therefore, C intersects every great circle of the sphere all the "melon slices" for all the elements of length of C must together at least fill the sphere. By adopting some limiting argument we get that the total length of C is (allowing for overlaps) $\geqslant \pi$, and this minimum will only be attained when C is a half great circle (from which some points may have been removed to their diametrically opposite points) or differs from such a curve in only trivial details.

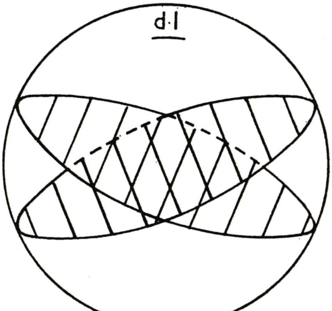
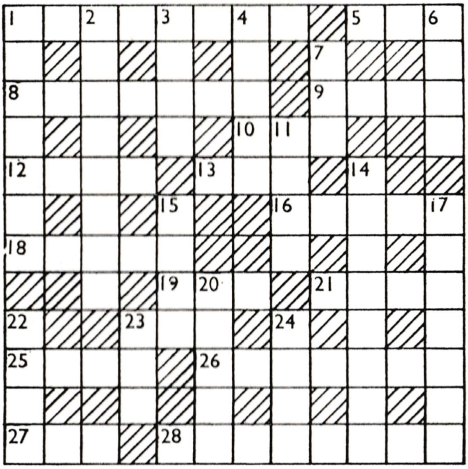


FIG. 3.

Crossword Puzzle

by

G. J. S. Ross



Across.

1. Extract velocity from a wave form: that's an easy job (8).
5. Division defines a number (3).
8. Constructed taking compass only, joining points (7).
9. Nothing important, a subscript perhaps (4).
10. Put that constant in, it's a trap (3).
12. In point of fact it suggests endomorphism (4).
13. No change for the collection? That's hard (3).
16. Nearly three, and breaking the law. There's a Mathematician for you! (5).
18. One of the Pools gets all the permutations (5).
19. First three terms of a geometric series, that's 21 (3).
21. Take away two, do you see? No (4).
23. Section of a cone (very singular!) (3).
25. Rouse him for recreations (4).
26. The French situation? That'll need another Mathematician (7).
27. His groups continuous? Don't believe it! (3).
28. Differential triangle? Not at all (8).

Down.

1. To find the gradient, differentiate log sin p (7).
2. This is purely symbolic (8).
3. A hundred years in prison! (4).
4. The maximum distance of the projectile is one chain (5).
6. Constant volume on parallel lines. That could describe an ellipse (4).
7. Opposite over Hypotenuse? That's very wrong (3).

11. A particle of infinite mass! (4).
14. His axiom was strictly Euclidean (8).
15. The length of the bridge? Nine inches (4).
17. Peer of Philosophy (7).
20. A former Archimedean patron (5).
22. A Mathematician that is ten times as noisy (4).
23. Golden section? That's ancient (3).
24. Collapse finally? That's extreme (4).

A prize of 10s. is offered for the first correct solution to reach the Editor.

A Fable

by

F. M. Hall

Once upon a time there were four friends: a scientist, an economist, an historian and a mathematician. They had been friends at school, and went up to Cambridge together, all determined to make their mark there.

For the scientist life was easy. With lectures and practicals and supervisions his days were filled, and he soon lost himself in the enormous amount of work to be done, having no time for much else. As he had never experienced anything different, he was very happy, worked well and was fairly certain to gain a good degree. The economist, on the other hand, concentrated on sport. He just missed his rowing blue, but got ones for soccer and cricket. He was always very tired after his athletic activities, but found that he could work at his economics without concentrating too much, and being naturally intelligent did not foresee any difficulty in his examinations. The historian was a social type. He sat on numerous committees, and more or less managed several flourishing societies, even becoming President of the Union. Of course this left no time for work, at any rate in term time, but his society activities helped him with his essay writing technique, and as he did manage to read one or two history books in the vacations he thought that he would be all right.

But the poor mathematician could not settle down. In his first year, fired by the example of his scientific friend, he tried to work all day long at his books. Such labour in mathematics is too much; by the end of the first term he had become thin and pale, and the year ended with him just avoiding a breakdown and his doctor insisting on three months' complete rest. So in his second year he tried to take more part in sport and started playing hockey. But he was so fatigued after his games that he found it impossible to concentrate on his work, and so learnt hardly anything during the whole year. For his final year he decided to learn from the historian, took part in several societies and was elected to one or two committees. Now, however, he had no spare time, and as his committee work could hardly help his mathematics, he again learnt very little.

The day came when the friends sat for their final triposes. The scientist knew his subject, and quickly worked right through his papers, answering without difficulty the questions set, which he thought were rather routine and very dull. The economist went along quite happily, guessing some things and making up others, and knew that with his natural insight he would gain good marks. The historian by now had a very polished style, and as he was careful to avoid mentioning facts which could be checked easily he too was confident of a happy outcome.

The mathematician was not so successful. He didn't know very much mathematics by this time, and he always attempted the wrong question, got stuck and spent an hour or more hopelessly trying to see his way through before abandoning the attempt and passing on to another problem. A few questions he did answer, or so he thought, only to discover shortly afterwards that slight mistakes at crucial points had completely invalidated his reasoning. He had slight hopes of much success, and was very depressed for some days after.

At last the time arrived when the four friends left the quiet courts of Cambridge and went out into the wide world. Now about this time there had been an abrupt change in the attitude of the various states towards one another. War had been abolished, all countries were quickly disarming, and everybody was afire with a feeling of world-wide brotherhood. The cessation of armament development produced a great surplus of scientists, with the result that our scientific friend found that he was unable to obtain work except as a washer of bottles, and by the time that scientists were again needed he had grown used to this, and would probably have been unable, even if he had wanted, to resume serious work. So he remained a bottle-washer, and ended his days at the top of his profession, as chief laboratory assistant in a fairly large laboratory.

The absence of research on warlike projects had resulted in increased drive to mechanise everyday life, and the economist found that work which he could previously have been involved in was now done by machine. He fell back on his sport, became a professional footballer, had to retire at the age of 40 and then started as a publican.

The historian had always intended a career in politics. He had visions of gradually coming to the forefront of his party, and ending as Prime Minister, or at least in the Cabinet. He thought that the times when his party was in power would amply compensate for those when it was out. However, he chose the wrong party. It was in opposition for most of the time, and our friend, at the age of 45 having become rather frustrated and therefore rash, quarrelled with his leader and ended his career where he had begun—on a back bench in the House of Commons.

Now how did our mathematician fare? He had scraped a third class degree, and hadn't much hope of a good job. However, the increase of automation had resulted in a great demand for men to work on the new computers and other elaborate machines which were being brought into use. Our friend was snapped up as a programmer by a very large industrial concern and soon became their head programmer, as the work didn't really require much mathematics, and he wasn't unintelligent. He and his firm discovered that he had great talents on the management side, and he gradually drifted away from computing and towards the executive. By the time he retired he had risen to one of the top positions in the concern, being also a director, and he continued to serve as a highly respected member of the board until his death at a great age.

The moral of this tale is left to the reader.

Traffic Jams

by

Anthony Bayes

The Highway Code, part 2, paragraph 18, states

> "Never drive at such a speed that you cannot pull up well within the distance you can see to be clear. Always leave yourself enough room in which to stop."

From the information given at the back of the Highway Code, under the heading "Vehicles cannot stop dead," we may deduce that if in dry weather a vehicle is travelling at x m.p.h., then the stopping distance in feet is $x + \frac{x^2}{20}$.

We suppose that the road can only take a single line of vehicles, and that each vehicle is y feet long. We define the capacity C of the road to be the number of vehicles per second capable of passing a given point on the road.

Clearly $$C = \frac{\frac{44}{30}x}{y + x + \frac{x^2}{20}} = \frac{44}{30}\,\frac{1}{y/x + 1 + x/20}$$

To find the maximum of C we find the minimum of

$$\frac{y}{x} + \frac{x}{20}$$

Differentiating and equating to zero

$$-y/x^2 + 1/20 = 0 \quad \text{i.e. } x = \sqrt{20y}$$

For simplicity we will assume that the average value of y is 20. Then x is 20 m.p.h.

In wet weather the stopping distance is doubled. Here

$$C = \frac{\frac{44}{30}x}{y + 2\left(x + \frac{x^2}{20}\right)}$$

The maximum value of C occurs when $x = \sqrt{10y}$. Hence in this case x is approximately 14 m.p.h.

I communicated these results to the Minister of Transport, and suggested that on busy roads the speed limit should be reduced to 20 or 14 m.p.h., depending on the state of the weather. As yet I have received no reply. . . .

Construction of Centre of Curvature on a Conic

by

C. C. L. Sells

Before giving details of the construction we first require a *lemma*:—

A point X on a conic and its Frégier point F harmonically separate the centre of curvature C and the mid-point M of the normal at X.

Proof: Take the tangent and normal at X as coordinate axes, so that the equation of the conic may be written in the form

$$ax^2 + 2hxy + by^2 + 2gx = 0$$

To find F, let any chord PQ of the conic be

$$\alpha x + \beta y + \gamma = 0.$$

Then the equation of the line pair XP, XQ is

$$\gamma(ax^2 + 2hxy + by^2) - 2gx(\alpha x + \beta y) = 0$$

and these two lines are perpendicular if

$$(a + b)\gamma - 2g\alpha = 0.$$

This is the condition that PQ passes through the point F

$$(-2g/(a + b), 0).$$

Now let C be the point $(c, 0)$. The circle, centre C, through X is

$$x^2 + y^2 - 2cx = 0$$

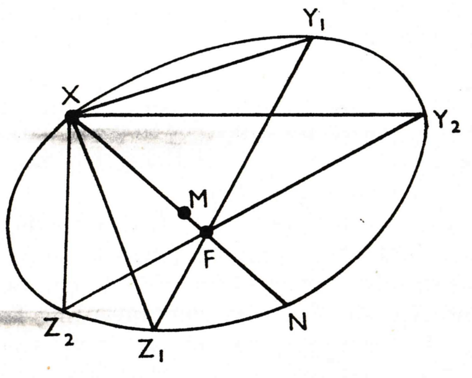
and this meets the conic where

$$4h^2x^2(2cx - x^2) = [ax^2 + b(2cx - x^2) + 2gx]^2.$$

For this to have three roots $x = 0$ the coefficient of x^2 must vanish, and so $c = -g/b$.

Finally the normal is $y = 0$, and this meets the conic again at N $(-2g/a, 0)$. Hence M is $(-g/a, 0)$.

Therefore the lengths XC, XF, XM are in harmonic progression and $(XF; MC) = -1$. Q. E. D.



The construction is now simple. Take two points Y_1, Y_2 on the conic distinct from X and each other. Let Z_1, Z_2 be the points on the conic such that $XY_1 \perp XZ_1$ and $XY_2 \perp XZ_2$ and join Y_1Z_1 and Y_2Z_2 to meet in F. Let XF meet the conic again at N and bisect XN in M.

Finally obtain the harmonic conjugate C of M with respect to X and F by the usual construction. Then C is the centre of curvature at X.

A Shorter Short History of Mathematics

by

FLOWER

(with apologies to Messrs. Sellar and Yeatman)

DEDICATION

I SHOULD like to dedicate this work to the dons—none of whom have inspired me to write it.

HISTORY

History, in general, is a bad thing, but the world situation today seems to indicate that there won't be much more and thus we deduce that the world situation is a good thing.

Mathematics is something that no one understands, hence it must be a good thing, and a history of mathematics is a good thing as I am writing it.

Mathematics begins with the concept of number. Number was first discovered with the birth of twins and the word "plural" coined to account for this phenomenon. For centuries historians and mathematicians (or vice versa) have argued which race first had twins—Greeks, Romans, Men, Chinese, Women or the Etc. Personally after many hours research in the "Horse and Groom" I favour the Welsh, partly because they're good darts players and partly because no one seems to have thought of them before, and anyway they seem such nice people.

With the discovery of plural, mathematics made great strides; one, two, three and many other numbers soon appeared. The credit for most of this work must be given to the Babylonians, as the Greeks and Romans were too busy inventing Latin to worry about maths, and the Egyptians were too busy chasing a fellow called Moses in and out of the Red Sea. In passing one must mention that Moses was not a communist: it was just that the Egyptians were highly moral and objected to bullrushes. However, the Egyptians quickly tired of chasing him as it became noticeable that more Egyptians went into the Red Sea than ever came out, so in the end they devoted their attentions to the Pyramids. They had had plans of these in mind for a long time and were only waiting for someone to discover the right angle before they started. At great expense a gentleman called Pythagoras (a Greek or Roman who wasn't interested in inventing Latin as he claimed that it was a bad thing and would, in time, kill all the Romans. He was right, too!)—at great expense Pythagoras obligingly discovered right-angled triangles and the way to the pyramids was open!

The next momentous discovery was made by the Arabs who were really Egyptians but didn't like the fact to be generally known. They reasoned, quite logically, that one day logarithms would be invented. Logarithms would require bases—so the Arabs invented the bases.

But the ingenious Arabs not only invented bases but invented the base 10 for natural numbers because this was helpful to the French in inventing their coinage system. The British, perpetually fearing invasion, resorted to the subtle ruse of counting in 12's, thus hoping that foreigners who were unable to count in 12's would be discouraged from invading Britain.

Time passes and the mystic art of mathematics comes to a standstill. Henry VIII tried to revive it and persuaded most of England to count up to six, but after that he gave up the ghost.

One cannot pass the Renaissance period without mentioning Leonardo da Vinci and Galileo. These were two of the most misguided souls in history. Galileo spent his time belting up and down the leaning tower of Pisa and bunging cannon-balls off the top—some historians reckon this was done for the benefit of American tourists, but as these hadn't been discovered yet one is forced to think that some historians had been drinking.

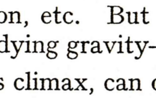
Leonardo da Vinci (Leo to his friends) was a bit of a clot as he tried to invent the helicopter before the Ministry of Civil Aviation had been founded by Lord Brabazon.

There remains one more great mathematician whom we have not yet mentioned—Sir Isaac Newton (known as Izzy).

Newton was educated at Cambridge (all the *best* people are), he became fairly rich (all the *best* people are) and had two parents (very rare amongst *best* people). He discovered one or two odd trivialities like Newton's laws of motion, etc. But his greatest triumph was when he was engaged in studying gravity—he discovered the apple!

And with this tremendous climax, can one say more?

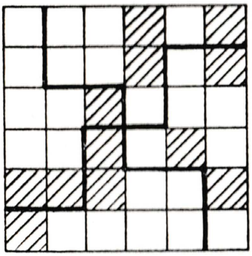
Solution to Elevation Puzzle

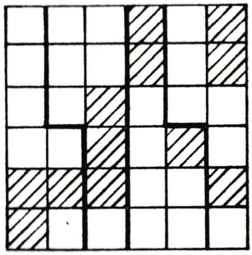


Solutions to Problems Drive

1. (i) 281 from cubic $n^3 - 4n^2 + 3n + 1$.
   (ii) 67 of p_n is the n^{th} prime, the series is p_{p_n}.
   (iii) (9,8) pairs are digits from π and e, alternately in that order and the reverse.
   (iv) 6 series is $p_{n+1} - p_n$.

2. Two solutions are:—





3. 5, giving remaining pieces of 6, 12, 24, 48 and 87 links.

4. 498 nuts: Peter had 60, Quentin 76, Ralph 135 and Stephen 227.

5. Michael and Mary.

6. (i) $f^2g^2 = f(fg)g = fg^3fg = fg^6f = fg^2f = g^3fgf = g^6f^2 = g^2f^2$.
   (ii) Can be proved by a similar argument.

7. (i) Number of seconds in a year of 365 days.
   (ii) Digits of π^2.
   (iii) 2^{50}.
   (iv) Recurring part of 1/17.

8. The first two are possible (the second easy); the third is not.

9. $(x,y) =$ (i) (8,19); (ii) $(-1,-2)$; (iii) (102,61); (iv) (455,1131) are possible solutions.

10. (i) $100 = (1 \times 2 + 3) \times 4 \times 5$.
    (ii) $3\frac{1}{7} = (-1 + 23)/(\sqrt{4} + 5)$.
    (iii) $32769 = 1 + 2^{3!+4+5} = 1 + (2 \times 3 + \sqrt{4})^5$.

11. 37.

12. If the first prime, p, leaves remainder 1 or 4 on division by 5 then $p^2 + 24$ is divisible by 5.
    If p leaves remainder 2 or 3 on division by 5 then $p^2 + 16$ is divisible by 5.
    If $p = 5$, $p^2 + 24 = 49$ is not prime.

Book Reviews

Topology. By E. M. PATTERSON. viii, 128 pp. (Oliver and Boyd, 1956.) $7\frac{1}{2}$ in. 8s. 6d.

Topology is a relatively new subject, but by no means a small one, and to write an introductory book of this size with no serious gaps is an impossible task. Dr. Patterson's book gains by being very readable, and by giving a rough idea of the methods used in each branch of the subject; it loses by being too slight to satisfy and by having no room to mention some of the tools (e.g. the exact sequence) which most characterise modern topology.

Fifteen pages are well spent on an enjoyable illustrated introduction which revives the usual topological chestnuts (Klein bottle, Four Colour Problem, etc.). Two chapters on Analytic Topology follow: Euclidean and metric spaces lead up to the general topological space and such ideas as compactness and convergence are well explained. Theorems proved in standard books on Analysis are sensibly quoted but one important theorem—Urysohn's Lemma—is proved, and represents the highest level of achievement reached in the book.

The second half of the book is an introduction to Algebraic Topology. Although it needs some of the previous results this is almost a separate subject characterised by the concepts of Homotopy and Homology. The chapter on Homotopy proves no startling result but succeeds (in the section on paths) in being unnecessarily complicated. The two chapters on Homology are altogether better value and an excellent introduction to the simplicial theory. Dr. Patterson is just going really well when small print and hurried pages proclaim that the end is nigh, with no space to hint at the good things to come or fully to apply his results to the questions raised in the introduction. It should also be mentioned that fallacies have been pointed out on pages 12 (Euler's Theorem) and 62 ("The product of two compact spaces is compact").

An author of Dr. Patterson's ability and a subject as important and varied as topology should have been allowed two volumes of this size. Nevertheless, as a stimulating introduction, which makes all but the most apathetic reach impatiently for the tomes listed in the bibliography, it is quite excellent. R. SCHWARZENBERGER.

Lie Groups. By P. M. COHN. viii, 164 pp. (Cambridge Mathematical Tracts no. 46, 1957.) $8\frac{1}{2}$ in. 22s. 6d.

This tract provides a much-needed introduction to a large and complicated subject. The opening chapters discuss the concepts of analytic manifold and of topological group, and the author then develops systematically and thoroughly the complete theory of the relation between local Lie groups and Lie algebras. A final chapter hints at underlying topology by expounding the elegant theory of covering groups.

It is probably wise in such a book to treat one aspect of the subject in full detail and to ignore others, but it is well to be aware that this has been done. Neither the topology of Lie groups nor the theory of Lie algebras is treated—but then I know of no book in English in which they are. Also analyticity is assumed from the start rather than, as is usual, being deduced from weaker hypotheses. Instead, there is a fair proportion of basic differential geometry, whose inclusion is necessitated by the present neglect of this subject in England.

Thus this book is an excellent introduction to this very interesting subject, but those stimulated by it to interest in deeper aspects of the theory will have to consult other works. C. T. C. WALL.

The Hypercircle in Mathematical Physics. By J. L. SYNGE. xii, 424 pp. (Cambridge University Press, 1957.) $9\frac{1}{4}$ in. 70s.

Despite its forbidding title, this is not a book of which applied mathematicians should be afraid. It is a lucid exposition of a systematic method of finding approximate solutions to boundary value problems in partial differential equations.

The method entails treating geometrical problems in function space. A function space is a space of functions defined in some domain of a physical space. Professor Synge develops the geometry of function spaces *ab initio*, not in the manner of a pure mathematician rushing from abstraction to abstraction, but slowly and clearly and making full use of geometrical intuition. The latter is very useful, despite the infinite dimensionality of function spaces.

Many boundary value problems in partial differential equations imply the geometrical problem of finding the point intersection of two orthogonal linear subspaces of a function space. This is done approximately by taking test functions, that is, test vectors in the two subspaces. For instance, if $\nabla^2 u = 0$ in a domain V with boundary Γ on which u is given, one subspace consists of vector functions with zero divergence in V and the other orthogonal one of gradients of functions which satisfy the boundary condition on Γ. For both subspaces the linear combination of test vectors which gives the nearest point to the intersection is found. This intersection, which is the required solution, lies inside a hypercircle with centre the midpoint of these two points (the approximate solution) and radius half their distance apart. (A hypercircle is an entity in function space corresponding closely to a circle). This accurate estimate of the error is the great advantage of the method.

The book contains a large number of worked examples with detailed calculations, the majority concerning Laplace's equation. Any second order self-adjoint linear equation can be solved by this method, which can be extended to systems of linear equations with several unknowns and derivatives of any order.

The short and rather unsatisfactory final section of the book deals with vibrational problems. The method can still be formulated, but difficulties arise as the metric of the function space is indefinite, and so a vector can have zero length without being zero.

However, the hypercircle method, of which Professor Synge was joint originator, is both a useful tool for applied mathematicians and a thing of mathematical beauty. This book can be strongly recommended to all who have to deal with boundary value problems in partial differential equations. Above all, the book has that quality so rare in mathematical textbooks, readability. C. HUNTER.

Special Functions of Mathematical Physics and Chemistry. By I. N. SNEDDON. viii, 164 pp. (Oliver and Boyd, 1956.) $7\frac{1}{2}$ in. 10s. 6d.

This is a welcome addition to a well-known series of University Mathematical Texts. It is intended primarily for students of applied mathematics, physics, chemistry or engineering who wish to apply the functions named after Legendre, Bessel, Hermite and Laguerre. Endeavouring to establish the most useful properties of these functions in a simple, compact form, the author has practically ignored complex variable methods. While this makes the book more intelligible to those with limited mathematical background, it is at the expense of more elegant procedures.

After giving a short introduction to the method of solution of a differential equation in series, the author treats the hypergeometric functions fairly fully. The properties of Legendre polynomials and associated functions are developed from the generating function. Further chapters are on Bessel functions (including a brief mention of modified Bessel functions) and the functions of Hermite and Laguerre. Examples are given of the application of these functions in classical physics and quantum theory. Concluding the book is a tit-bit on the Dirac delta function. The examples given at the end of each chapter should be very useful for extending the material given in the text.

The format of the book, within the limitations imposed by the small page-size, maintains the high standard set by previous Oliver and Boyd texts. A fairly thorough reading revealed some two dozen misprints in the text, but the proof-reader who has to check so many sub- and super-scripts is not to be envied. But these are trivial considerations; the book fulfils the limited purpose for which it was written, and should find a place on many bookshelves.

D. A. NIELD.

A German—English Mathematical Vocabulary. By S. MACINTYRE and E. WITTE. xi, 95 pp. (Oliver and Boyd, 1956.) $7\frac{1}{2}$ in. 8s. 6d.

Among the recent spate of German grammars for scientists, it is most welcome to find one directed principally to the pure mathematician. The first fifty pages contain a German-English dictionary—approximately 2,000 words, equally divided between exclusively mathematical terms and other basic expressions likely to be found in a technical text; applied mathematics, statistics, and logic are specifically excluded. The remainder of the book is devoted to a grammatical sketch, succinctly and lucidly presenting parts of speech, word-order, declensions and conjugations, each section illustrated by copious examples in suitable vocabulary. Other short but notable features are lists of abbreviations, numerals and (inevitably!) strong and irregular verbs; also a very useful table of gothic type and script, and two specimen pages of translation. The printing is even better than that of companion volumes of the University Mathematical Texts.

While this little work greatly facilitates translation, it by no means obviates the necessity for an ordinary dictionary, and many words are omitted which should fall within its compass: the reviewer confidently anticipates a much enlarged and more helpful second edition.

H. T. CROFT.

Convexity. By H. G. EGGLESTON. viii, 136 pp. (Cambridge Mathematical Tracts no. 47, 1958.) $8\frac{1}{2}$ in. 21s.

The aim of this tract is to provide for mathematicians, economists and others wishing to use or to understand the idea of convexity an introductory account of the subject combining generality with simplicity. In many ways the author achieves his aim. The account is well written, well printed and takes the reader to the bounds of the field in some directions. But as the reader is assumed to be familiar with a fair amount of analysis and the simpler concepts of topology, one wonders how many economists will appreciate it!

A set is convex if given any two points of the set, the segment joining them is also part of the set. The placing of this restriction on sets simplifies the analysis of many problems and gives rise to a number of interesting properties.

In the first five chapters the basic properties of convex sets and functions are developed. These ideas are extended and further developed by using classes of convex sets. Certain operations on classes of sets are also studied.

Having thus done the spade work and proved a number of important theorems, Dr. Eggleston demonstrates the theory by proving some interesting results in the last two chapters, descending where necessary from n-dimensions to two. Here are two proofs that of all convex sets with a given volume the sphere has least surface area, Professor Besicovitch's result that in two dimensions the most asymmetrical convex set is a triangle, and a number of inequalities relating the circumradius of a set to its diameter, and the inradius to its minimum width. The last chapter contains the analogous results for sets of constant width. The author points out a few unsolved problems and gives a brief account and bibliography of other books on the subject.

This book is worth reading not only for its interest but also for the clarity with which the author explains both what he is trying to do and why. These explanations are relevant in places not merely to the matter in hand but to mathematical technique and method in general.

M. FIELDHOUSE.

An Introduction to Fourier Analysis and Generalised Functions. By M. J. LIGHTHILL. viii, 80 pp. (Cambridge University Press, 1958.) $8\frac{3}{4}$ in. 17s. 6d.

The dedication of this book is: "To Paul Dirac who saw that it must be true, Laurent Schwartz who proved it, and George Temple who showed how simple it could be made." Professor Lighthill gives a rigorous definition of those "improper" functions used with such abandon in modern mathematical physics; a simple definition based on such elementary analytical tools that one wonders why it is not slipped into the Tripos course somewhere. In this generalisation of the notion of function Hadamard's finite part and the Cauchy principal value of an integral occur naturally. Fourier series are treated as rows of delta functions, and are always convergent and differentiable term by term provided only that the n^{th} coefficient is $O(|n|^n)$. A chapter is devoted to the asymptotic estimation of Fourier transforms. It is a readable book with ample examples, recommended to pure mathematicians as well as those who have been taught: "Delta functions! Oh, someone has made them respectable."

F. P. BRETHERTON.

The Theory of Ordinary Differential Equations. By J. C. BURKILL. x, 102 pp. (Oliver and Boyd, 1956.) $7\frac{1}{2}$ in. 8s. 6d.

A good English textbook at undergraduate level, on the theory behind the standard forms of solution of differential equations, has been needed for some time, and this book has obviously aimed to fill the gap. From the point of view of Cambridge Part II mathematics students, at least, it has succeeded; but I doubt if many engineers or scientists (to whom reference is made in the Preface) will have the necessary background of mathematical analysis to follow the rigorous arguments or the full use which is made of complex variable theory.

The existence theorem for solutions of $y' = f(x,y)$ is proved in the first chapter, and possible extensions of the theorem are indicated. A short, but fairly comprehensive, account of the linear equation follows; and then a rather more difficult chapter entitled "Oscillation Theorems," containing several results on zeros and distribution of solutions. The next three chapters give an excellent account of methods of finding solutions as infinite series and integrals. Then follow a short section on Legendre functions; another (more useful) one on Bessel functions; and a final chapter introduces asymptotic series. The virtue of the last three chapters is that they illustrate the ideas of the first part of the book, which in places appears to be an abstract and rather confusing list of theorems with no apparent application. For this reason it should be more use as a reference than as a complete introduction to the subject.

A. J. HEARNSHAW.

Books Received

The following books have also been received, and reviews will appear in our next issue:

An Analytical Calculus Vol. IV.
By E. A. MAXWELL. xiv, 288 pp.
(Cambridge University Press, 1957.) $8\frac{3}{4}$ in. 22s. 6d.

Toeplitz Forms and their Applications.
By ULF GRENADER and GABOR SZEGÖ. x, 246 pp.
(University of California Press, 1958.) $8\frac{3}{4}$ in. 45s.

Ordinary Difference-Differential Equations.
By EDMUND PINNEY. xii, 262 pp.
(University of California Press, 1958.) $9\frac{1}{2}$ in. 37s. 6d.

Multivalent Functions.
By W. K. HAYMAN. viii, 152 pp.
(Cambridge Mathematical Tracts no. 48, 1958.) $8\frac{1}{2}$ in. 27s. 6d.

Cambridge University Press act as agents for the University of California Press.