QARCH

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1 Rooks on Chessboards. A chessboard has as many columns as rows but differs from a regular chessboard in that the distribution of black and white squares are random, provided that there is at least one white square in each column and that there is at least one whole white column. We have an unlimited amount of rooks to our disposition, so that there will always be enough. We say that we aligned the rooks successfully on the chessboard if we satisfy all of the following conditions:

(1) rooks stand only on white squares

(2) there is at least one rook on the chessboard

(3) the rooks do not attack each other

(4) each white square not occupied by a rook, but horizontally attacked by a rook is also vertically attacked by some rook.

Prove that it is always possible to assign rooks according to 1.,2.,3.,4.

2 Random points. There are n randomly chosen points in the plane, no three of which lie on the same line. Is it always possible to find a closed n-gon with non-intersecting edges whose vertices are the n random coplanar points?

3 Colourings. A regular dodecahedron can be coloured with 4 colors in such a way that neighboring faces differ from each other in colour. Prove that there are only 4 different colourings obtainable in this way.

Remark: We consider two regular dodecahedra as identically coloured if they differ from each other only by a rotation.

4 Triangle nets. As it is known, the whole Euclidean plane can be covered by a net of equilateral triangles.

Is it possible to put one of the signs '+' or '-' on each of the vertices of the net in such a way that in each of the triangle-components, the following rule holds: if two vertices of a triangle have the same sign, then the third vertex has a '+' sign, and if two vertices of a triangle have opposite signs, then the third vertex has a '-' sign.

Of course, it is possible to put '+' everywhere, but this trivial solution is excluded from considerations.

5 Triangle division. Divide a triangle into 19 sub-triangles in such a way that there is the same amount of edges stemming from each vertex of the newly created figure (also in the original big triangle). In this problem, the number 19 is not allowed to be replaced by any number bigger than 19, but it is allowed to be replaced by numbers smaller than 19. What are these numbers?

Solutions

Please send in your submissions (full or partial solutions, new problems, comments or queries), to

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or to the Archimedeans' pigeonhole at the CMS. Items sent by post should be addressed as follows:

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There is very little restriction on the type of question suitable for publication and discussion in these pages, except that the statement and (if known) the solution be relatively concise. Problems may be submitted with or without known solutions.